

---

---

# THE SELF-GENERATING PRINCIPLE OF QUADRATIC RECIPROCITY

by Pierre Beaudry 5/22/2020

---

---

## FOREWORD

Geometry, Arithmetic, Music, and Astronomy (GAMA) were the four domains of knowledge that Pythagoras inherited from the ancient Egyptian science of Sphaerics, which he developed in the form of a *Quadrivium* program for the education of children in ancient Greece. This is how Archytas, Socrates, and Plato, among many others, were educated in order to become Promethean creative thinkers. Pythagoras chose those four domains of knowledge as the basis for all knowledge because they were each in its own way oriented toward the future.

However, the Greek Oligarchy centered in the priesthood of the Oracle at Delphi saw this as a threat to their power and subverted the Pythagorean project, condemned Socrates to death, and sold Plato as a slave. Why? Because this science of Promethean thinking was oriented toward the future; that is, toward creativity; and if creativity could be curtailed and the population controlled, then, Oligarchism could never be defeated.

The question therefore is: How can mankind retrieve this ancient method of mental development and revive the powers of developing the mind to a higher manifold? The way to achieve that is by defeating today's British oligarchical way of thinking by deception. This can be done by adopting the LaRouche-Riemann method of increasing the energy flux-density, which I intend to introduce, here, by means of a constructive form of thinking. Lyndon LaRouche often said: "Believe nothing that for which you cannot give to yourself a constructive proof." I believe that this was also what Leibniz meant to develop with the method he called *analysis situs*.

## INTRODUCTION

“I say to the tyrants of the world: ‘I stand here like Prometheus. I despise you! And I’m trying to make men in my own image, who despise you, as I do!’ And that’s what it’s all about.”

Lyndon LaRouche, Lyndon H. LaRouche, Jr., [The LaRouche Youth Movement: International Strategy To Build a Bridge to the Future](#), EIR, October 19, 2007, p. 71.

In a recent email correspondence with Fred Haight, I discovered a connection between Brahms’ 4th Symphony and the underlying principle that Gauss had discovered in his scientific research on whole numbers, and that such a principle also corresponded to what Lyn had proposed the LYM to work on in The Basement group, during the 2007-2010 period, on the subject of discovery of principle of the “*ontological infinitesimal*.” Here is the puzzle that Fred provoked me with on April 3, 2020:

“In preparation for our next exchange, I suggest you start listening to Brahms' 4th symphony. Why Brahms, when Bach, Mozart and Beethoven are so far superior? Like the Gospel of John, it was born out of combat, the necessity to refute falsehood. That can lead to a bare-bone, bare-knuckled approach.

“Bruckner's symphonies expressed musical diarrhea. Brahms called them ‘symphonic boa constrictors.’ Bruckner's followers felt free to chop out 20 minute sections. No-one missed them. Brahms wrote that not one note could be added or subtracted to his symphonies, without detriment to the whole.

“Here is the first movement. The theme is 2 notes down, B-G, a major 3rd, and 2 notes up, E-C, a minor sixth, the inversion. That's all! It derives from Beethoven.”<sup>1</sup> <https://youtu.be/1xZcBqzdhQ8>

Upon receiving this message, I immediately heard, in my head, a series of self-generating harmonic relationships forming a spherical cycle which started to whistle in the silence of my mind, as if it was coming from the future of the Universe itself. So, I went to the keyboard to play what my mind was whistling. The notes started turning on themselves as if they were rotating from a pre-established order inside of a sphere.

As soon as I played the quadratic set of those first four notes on the keyboard, **B-G** and **C-E**, which appeared as a sort of inversion, I heard the following series of sixteen notes which came calling each other into existence, one after the other, in the following cyclical order, four by four, as if they had been generated through some invisible Lydian spiral beginning with **B-G** and ending with a different **B-G** relationship. This is what I heard:

**B-G** and **C-E**  
**A-F#** and **D-B**  
**F-D** and **C-A**  
**F#-E b** and **B-G**

What was the meaning of this? I didn't have a clue, until I started looking back into Leonardo da Vinci's geometry of knots. There, I discovered a sort of map to travel through, like Leibniz's *pre-established harmony of analysis situs*; that is, the same principle that John Sigerson identified in his presentation to the Schiller Institute on April 25-26, 2020, where he suggested a sort of empathy (*Einfühlun* or *empfindlichkeit*) of the next notes that one can hear coming from the future, as if from a pre-established ordering in the universe. During his presentation, John referred to the idea of “preexisting notes” that Lyn had discussed with him and Mindy Pechenuk during a visit they had made to the Minnesota Rochester prison, in January 1993. Lyn said:

---

<sup>1</sup> Personal correspondence.

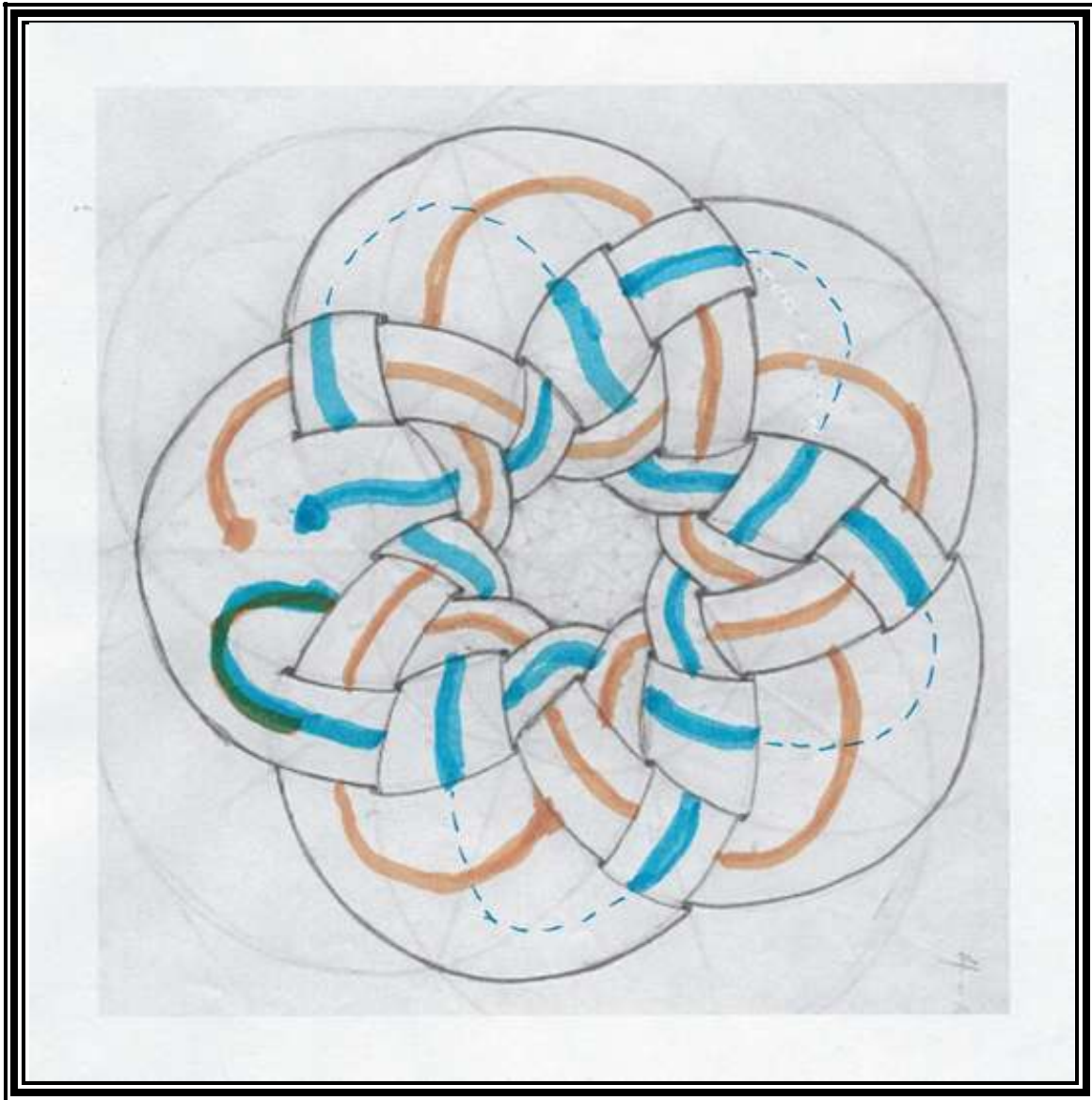
“First of all, the musical domain is a quantized field; notes exist, and space is Keplerian. Because you have the notes, they exist in certain locations, there are certain harmonics that exist, they’re ordered. And no matter what notes you’re playing, the next one is going to be there. You can change your sequence as much as you please, but the next one is going to be there. It’s all predetermined for you. And this is not alterable. And an approximation of the note, only to the extent that you’re not cheating, is the note. The note that is sung or performed is not the note. It’s the best approximation of the note. The tone is absolute; and the performer merely approximates that. And if they don’t approximate that rather well, we get unhappy; we get disturbed. But it’s analysis situs.”<sup>2</sup>

I realized that this pre-established harmony of the musical notes in physical space-time was a divine ordering upon which everything that existed in the universe had been created from a self-generating principle of quadratic reciprocity. This process is like going on a “No-Where”, as my father used to call it, while teaching us how to drive a car. It’s is not the apprehension of the destination that is supposed to keep us on alert, but the enjoyment of driving back from where you had never been, and finding your way without getting lost. The joy of a “No-Where” is in the discovery of how to go into the future and return safely from where you had never been before.

---

<sup>2</sup> Quoted by John Sigerson, [\*The Physical Power of Classical Poetry and Music\*](#), Morning Briefing for Tuesday, April 28, 2020, p. 16 of 26. See also: [The Schiller Institute Conference: \*How Human Creativity Will Solve the Present Global Crisis\*: \*\*Panel 1: The Urgent Need to Replace Geopolitics with a New Paradigm in International Relations\*\*](#)  
[Panel 2: For a Better Understanding of How Our Universe Functions](#)  
[Panel 3: Creativity as the Distinctive Characteristic of Human Culture: The Need for a Classical Renaissance](#)  
[Panel 4: The Science of Physical Economy](#).

## 1. THE GEOMETRY OF AN AMAZING GAME



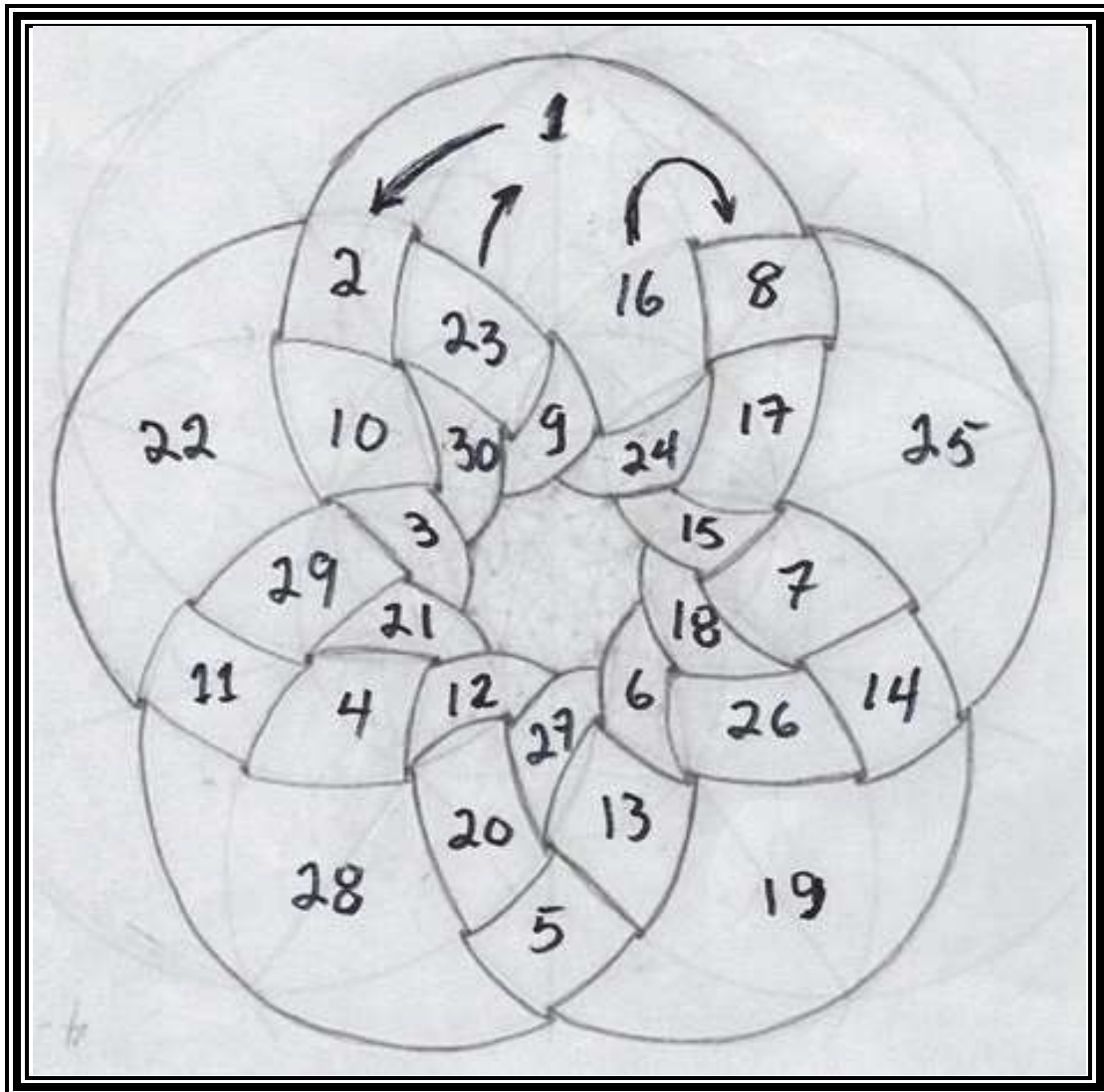
Knot Maze of the Golden Section

Take your cursor, and follow the two successive pathways (brown and blue) of the above maze in such a way that you return to your starting point without getting lost and without missing a turn. You are allowed to turn, go inside, under and over obstacles, but you are not allowed to go backward. As in a planetary orbit, you are allowed to go back only by going forward. Start with the brown and end with the blue pathway, or *visé versa*, and come back to where you started. I know it's a contradiction, but try it anyway. You can go back and change the way you started from; your mind is able to do that all the time.

These pathways are pre-ordered galactic pathways of cognition, because the universe is not based on particles but on pathways of change; the universe is not based on the periodic table of the elements, but on principles of change among such elements. The universe is based on the principle of cosmic radiation of waves which work like the mind works.

In a doubly-connected manifold, that is, on a flat surface, two opposite directions, such as clockwise and counterclockwise directions, cannot coincide, because their orientations are contrary to each other, and, according to the logic of Flatland, two contraries cannot coincide. However, that is not true inside of a triply-connected manifold such as this maze torus. You can easily convince yourself of this by constructing a Moebius strip in three dimensions.

Furthermore, when going through the maze, count your steps by leaving the footprints of ordinary numbers behind you. In this manner, you will be able to see how you can go in the two opposite directions without interruption.



You have taken thirty continuous steps only in the same continuous forward direction without any interruptions, clockwise and counterclockwise. You have also traveled forward and backward on different pathways and have come back to the same place a different person than you were when you started. The directions are changing and yet, they coincide, because they are folded together and the discontinuities of their opposition no longer exist. Think of this as an exercise in Cusa's solving the paradox of the *coincidence of opposites*.

You may also have discovered that contrary to moving on a flat surface, a three dimensional maze permits you to go under and over, inside and outside of

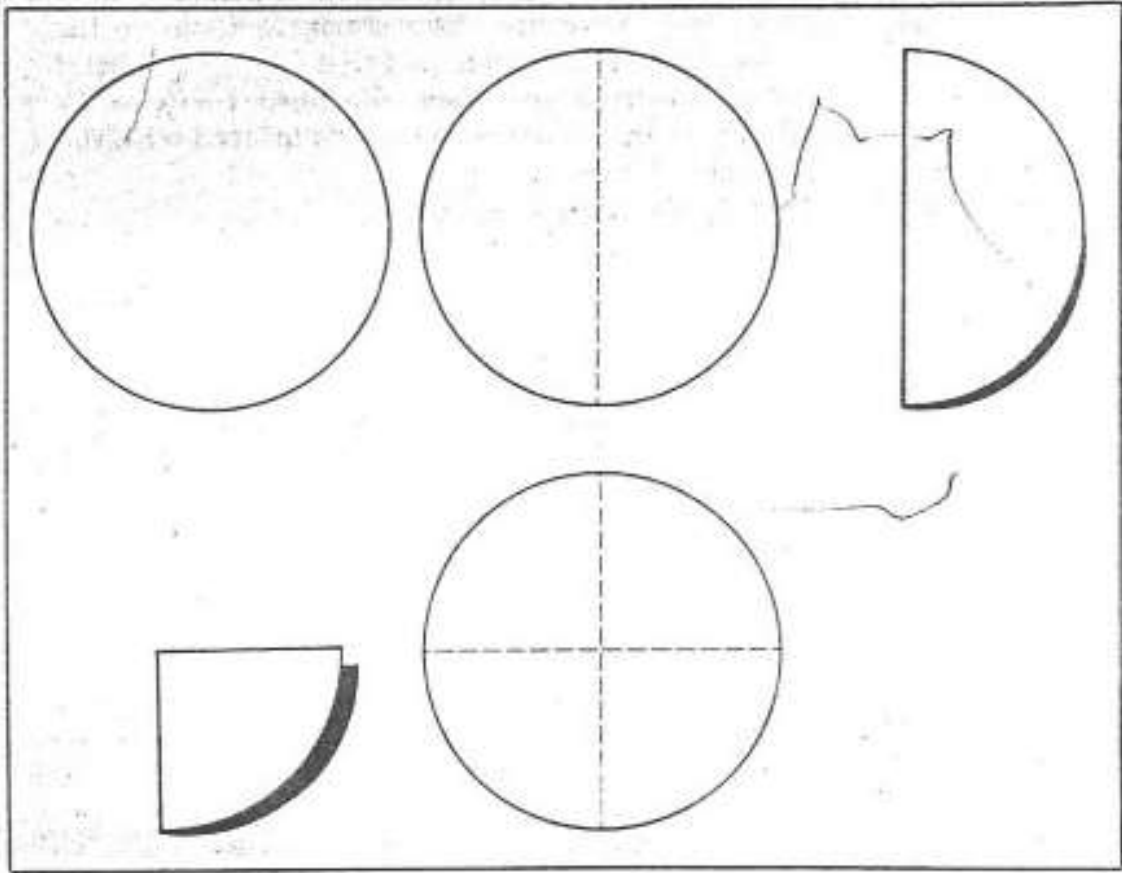
things, without bumping into boundary limits. You may also have discovered that by covering your own tracks with ordinary whole numbers, you have covered the entire space and have not left any empty space behind. That's *analysis situs* as Leibniz recommended constructive geometry should be.

The secret of this method is very simple. All you did was to go from a two dimensional domain to a three dimensional one; you got up from the Flatland where the British oligarchy has been keeping your mind in shackles for a very long time, and you went beyond to the [\*Lanternland\*](#) domain of Francois Rabelais. This is also what happens when you discover the virtues of a different human culture that you did not know before. You discover a new dimensionality.

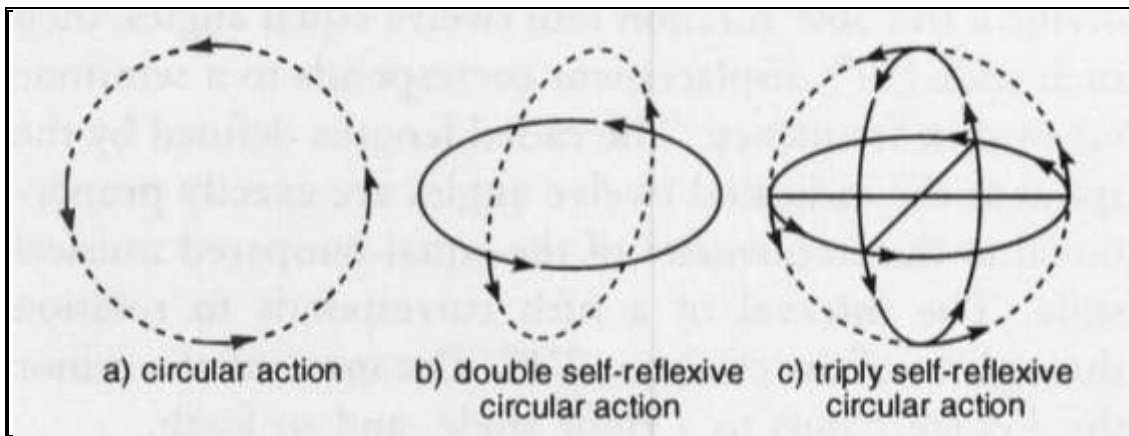
I say human culture instead of animal culture, because animal culture cannot change. If you keep the British type of cultural axioms that reduces human activity to animal behavior, you may never change either. For example, the point is not to look among each other for what is common between human beings and animals. Humans do not cooperate with each other in order to compete against each other, in order to survive like baboons do; humans cooperate in order to make the human species immortal. That's the secret of Leibniz's method that he called "*Pre-established Harmony*."

Take the following example of the polygon and the circle. Those are two different species as are animals and human beings. Don't attempt to find circular behaviors among polygons; there are none. That's a typical problem that opposed two different views of human culture in Ancient Greece; one was the view of Solon of Athens with the Pythagoreans, and the Platonists on the one side, and the opposite was the view of Lycurgus of Sparta with the Aristotelian cult of Delphi on the other side. The mortal conflict between them was to either follow the Promethean quest of immortality and of mastering triply-self-reflective circular and spherical action, or follow the rule of Zeus for leading the human herd to accept quietly how to go along to get along.



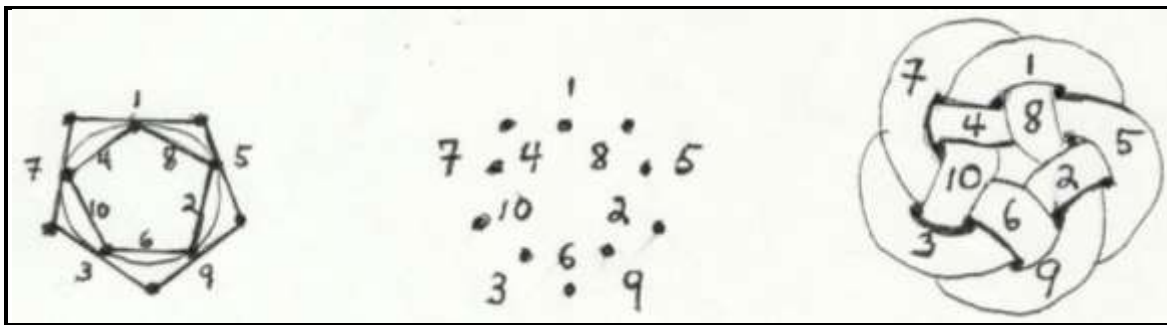


*The foundation of all geometry is circular action, that is self-reflexive, and generates the plane, the line, and the point.*



The British oligarchy and their mathemagicians have convinced most people that only mathematics can eliminate the opposition between the straight line and the circle by squaring the circle. Untrue. That's a way to fake a solution to a problem. They make you believe that if the side of a polygon circumscribing or inscribing a circle is small enough, it won't show, and it will become so close to the circle that you won't be able to make the difference. Nonsense! The difference will still be there, in your mind, because the two opposites are irreconcilable species which belong to two different worlds. How do you solve that problem? Jump to the next higher manifold.

The straight line and the curved line are axiomatically different; but they are opposites only in Flatland. If you go to Lanternland, the problem of their difference can be solved with Leibniz's principle of continuity.<sup>3</sup> Take a look at the following flat pentagons (one inscribed within the circumscribing one). How can these two pentagons (on the left) ever connect together? How can the inscribed pentagon go beyond the circle and join in a dance with the circumscribing pentagon? Just count the sides by self-reflexive rotation and see how it is impossible.



However, if you create a higher dimensionality you can jump from simply-connected circular action to doubly-connected and self-reflective circular action; that is, from the flat circle to the tubular torus knotwork. In other words, the two pentagons dance together by going around the torus twice in a single action; that is, by rotating and by orbiting at the same time. The “round” Earth that you stand on has been doing that double-motion all of your life and you don't even feel it. Next,

<sup>3</sup> Gottfried Leibniz, Specimen Dynamicum in, *Philosophical Papers and Letters*, Kluwer Academic Publishers, Volume 2, Second Edition, Boston, 1989, p. 447.

make a second jump and apply the same principle to the triply-connected manifold of Vernadsky.

Everything that is derived from polygons and the Five Platonic Solids is derived from the same principle as the Lithosphere, Biosphere, and Noosphere. However, the Abiotic domain is expressed by the Sphericalness of the Noosphere and not the other way around. As Lyn said:

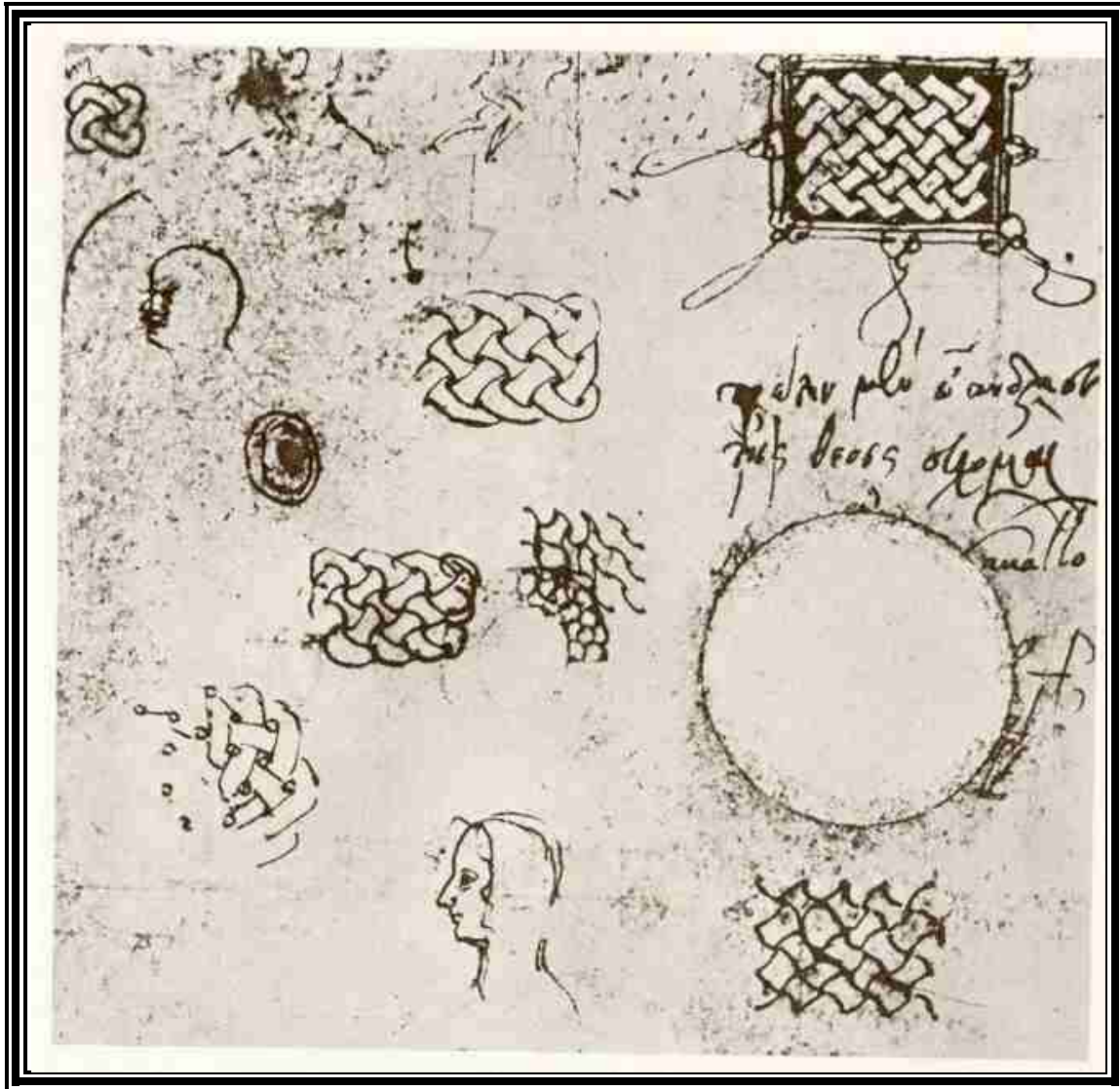
“The universe is organized as a process of development. The principle of mind exists in the universe. It exists from the beginning of the universe—whatever the beginning is, whatever the beginning means. Life, similarly, inferior to cognition, also lives in the universe, as a distinct phase, which is distinct from human creativity. And on the lowest level, you have the Lithosphere. As long as you get this crazy model, you’re always assuming backwards: You’re assuming that the Lithosphere is primary; that somehow you’ve got to figure out how the sawdust grew into something-or-other as a Biosphere; and then the Biosphere spun off the Noosphere. Crap! It’s the principles of the Noosphere that run the universe! So, the Noosphere was there in the beginning. Where is it located? Hmm...”<sup>4</sup>

However, when you consider the Galaxy from the vantage point of cycles of cosmic radiation, you have to develop a completely different hypothesis. The assumption, here, is that, ontologically speaking, as Lyn postulated, there is something common between galactic cycles and cycles of the human mind, both are expressed through a *self-development principle of noësis*. The axiomatic difference to remember is that the time and space scale levels are completely different from those of sense perception. Let’s try this new space-time experiment and see.

---

<sup>4</sup> Lyndon H. LaRouche, Jr., [LaRouche/Basement Team Dialogue: Mind Is the Principle of the Universe](#), EIR, Vol. 37, No. 41, October 22, 2010, p. 13.

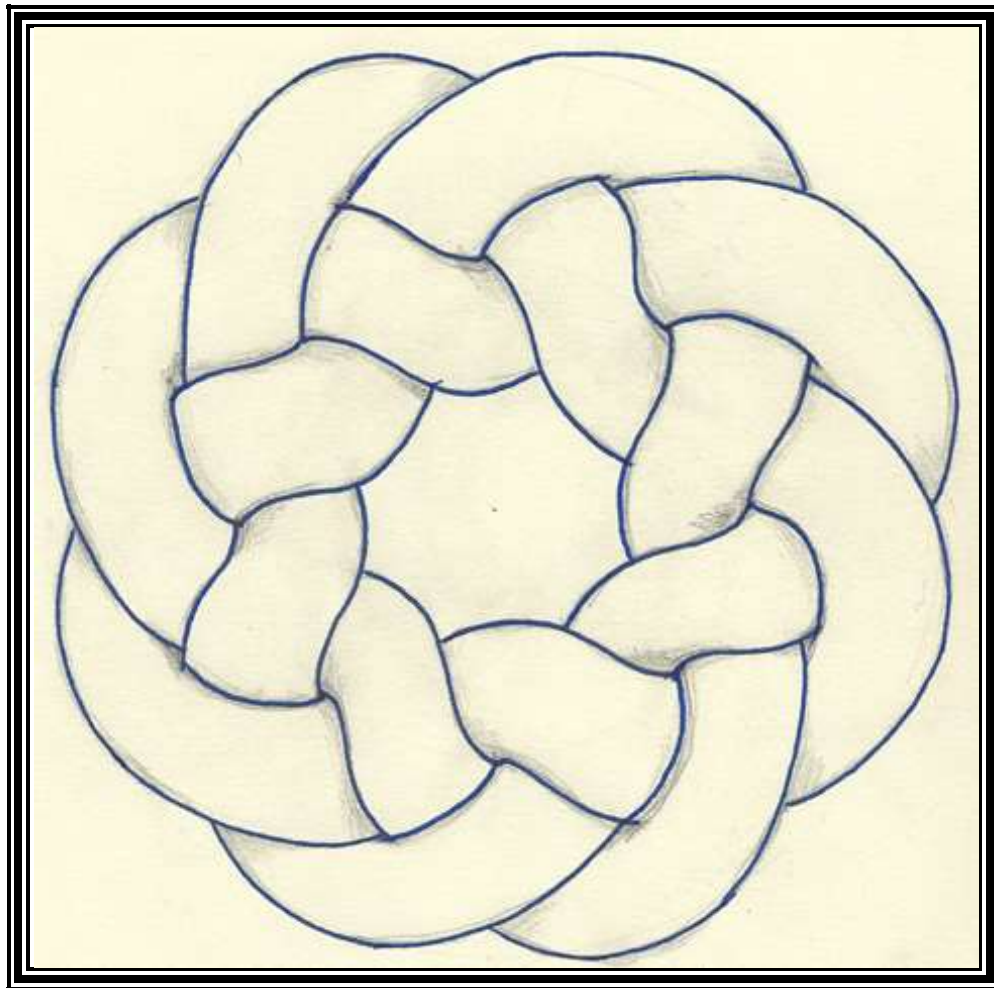
## 1. A QUADRATIC RECIPROcity INTERACTION



Leonardo Da Vinci knots.

Go through the following octagonal maze below by counting your steps in such a way that you go over or under one new space each time you take a new step, suggesting the presence of isochronicity of time which is attached to each action everywhere. The exercise consists in finding sixteen sets of reciprocals from 0 to 31. Your first and last number is 0, and all the numbers in between must be ordered in succession from 1 to 15; and then, you count from 16 to 31 back to 0. The

ordering of the numbers has to be in accordance with the following accelerating/decelerating trajectory pattern, as if you were going to Mars: 0, 1, 12, 123, 1234, 12345... up to 15; then from 16 to 31, back to 0, through ...12345, 1234, 123, 12, 1, 0. Place the 0 anywhere you wish on the knotwork below and then start either clockwise or counter clockwise to place 1, and then 12, and then 123, etc. up to 123456789101112131415; then, come back down from 16 to 31 back to 0.

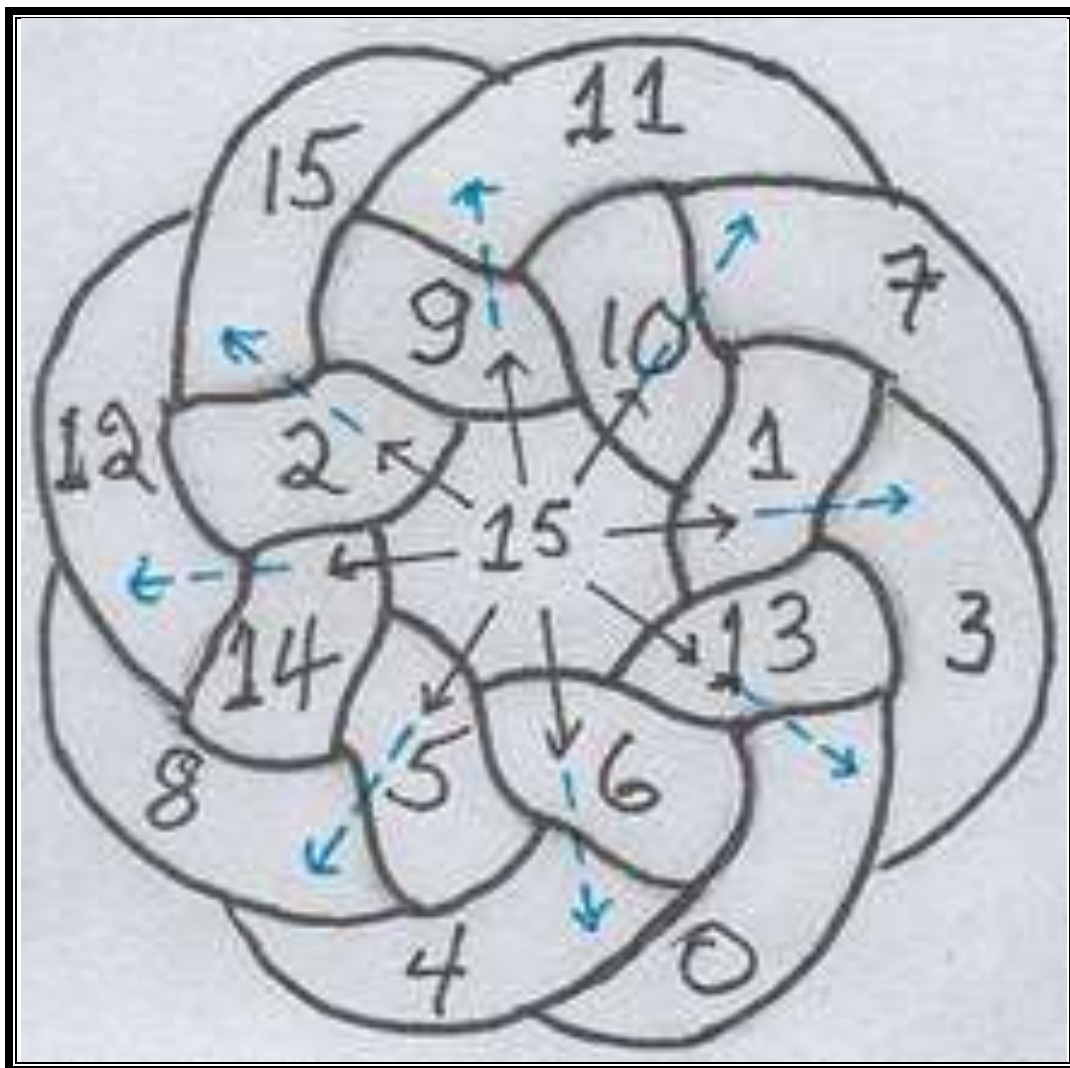


The analysis situs of the octamaze is based on accelerating from 0 to 15 and decelerating from 16 to 31, back to 0.

If you go through this maze without skipping a step, you will have gone through an experiment similar to Brahms's exercise that Fred sent to me. This octamaze is a little tricky, however, because it is one of those exercises in number

theory that Carl Gauss used to amuse himself with when he wrote *Disquisitiones Arithmeticae*, but which can be composed with a simpler geometrical method than the one he used. Gauss called his method: quadratic reciprocity; I call it *reciprocity in the simultaneity of eternity*. One of the most fascinating aspects of this quadratic octamaze is that the ordering of such an accelerating-decelerating process only works for the C-256 series.

### SOLUTION TO THE OCTAMAZE QUADRATIC RECIPROCITY



Octagonal reciprocity solution

After finding this solution, I realized that the Brahms composition had something to do with both Leonardo da Vinci's knots and Gauss's quadratic reciprocity; but, I had no idea why or how those four sets of quadratic reciprocal inversions connected together in such a cyclical fashion, or why they appeared to correspond to some sort of Pythagorean Quadrivium. That's when Lyn's idea of Sphaerics came to my mind. What is their bounding principle if not the sphericity of the quadrivium? Look at it in the following way.

As Einstein saw it, the curvature of the universe as a whole is such that the distance between things is equivalent to zero, because everything connects isochronically as well as reciprocally. This is what happens when you apply the principle of least time to space as Fermat did; that is, when you apply the principle of acceleration/deceleration in the simultaneity of eternity. Lyn gave the following hint for this conception when he discussed the following higher hypothesis:

*“The crucial distinction between analog and digital functions lies precisely, and uniquely in the mode of the notion of analog functions associated with the ancient Pythagorean quadrivium, the work of Plato (as in competent Christian theology), or that of Philo of Alexandria, for example, but never Aristotle or Euclid. (Emphasis added) This quality of creativity, which never appears in lower forms of life, is the only distinction of human behavior which separates the increase of potential relative population-density of the human species (and society) from the population potentials of the higher apes.”<sup>5</sup>*

From this point of view, therefore, my hypothesis became pulled upward to a quadrivium-spherical principle, which was the same as the one that Lyn had attributed to the ancient Pythagoreans and Plato, and to Keplerian astronomies, and which reflected Leibniz's *Pre-established Harmony*. Here is what Lyn wrote about the “quadratic reciprocity” question and about adopting a bounding principle of “sphericalness which related to this quadrivium.” As Lyn demonstrated throughout

---

<sup>5</sup> Lyndon H. LaRouche, Jr., [Science & Religion: Life at an Atheist's Funeral](#), EIR, Vol. 35, No. 4, January 25, 2008, p. 62.

his works, the only way that the digital system could work properly was within an analog quadrivium of reciprocity. I remind the reader, here, that Gauss had also been hiding this knowledge behind the mask of a modular wave function that Lyn deciphered as follows. I reproduce Lyn's text, here, in its entirety:

### **“QUADRATIC RECIPROCITY**

“This set of considerations obliges us to turn our attention to the most profound of the issues of the method required for scientific progress in general. On this present occasion, I take a broad step which is similar to what I have published on similar matters earlier, but is nonetheless a qualitative and also necessary step beyond what I have presented in related matters on which I have written earlier.

“From the work of the ancient Pythagoreans and Plato, through the crucial discoveries, as by Nicholas of Cusa, Leonardo da Vinci, Kepler, and Leibniz, as capped, thus far, by that of Riemann, Einstein, and Vernadsky, all actually competent insight into crucial matters of science, as since the design of the great pyramid of Giza, is always to be rooted implicitly in the subject of astrophysics. There is nothing merely coincidental in that choice; the choice is unavoidable. For those among us who are thinking clearly today, those relevant, better-known ancients, such as the Pythagoreans and Plato, used the concept of the “universal” to signify either the notion of the entire existence of the known, stellar universe, or a physical principle which could be implicitly attributed, pervasively, to be, functionally, a metrical characteristic of the whole interior of the domain of that universe, so defined.

*“At first impression, the starry universe appears to be spherical. Why is that so? Does that appearance not imply that a quality of “sphericalness” bounds the universe? If so, does something else, of a still higher epistemological authority, bound that apparently spherical quality of boundedness? These are not merely coincidental questions, but profoundly ontological questions to be treated by appropriate methods of investigation; these questions imply a different question of deadly seriousness: How was*



*this stubbornly persistent appearance of spherical boundedness, as by the Pythagoreans and Plato, generated for the mind of man?*

“Two great questions are implied in that set of questions. The first of these questions is expressed in the form of the elementary notion of an anti-Euclidean geometry of the type underlying the physical science of the Pythagoreans and of the related circles of Socrates and Plato. The second, deeper question, which is also implied in certain features of their work, as also the famous argument of Heraclites earlier, is: to what degree is the way in which we acquire reliable scientific knowledge, itself a reflection of the “systemic architecture” of what appear to be the specifically human biological conditions under which all valid human knowledge of the universe is organized? (Footnote: Compare this to my earlier emphasis on the fact of the way in which the biological design of the functions of the human mind define the way in which the “architecture” of the imagination is designed.)

“Thus, Kepler’s uniquely irreplaceable, original discovery of the principle of universal gravitation, has continued, in fact, to typify the proper modern use of the term “universal” to the present time.

“In the course of time, one member of the team working on Gauss’s discovery of the Ceres orbit brought up the matter of Gauss’s ominous remarks on the subject of quadratic reciprocity. Gauss’s emphasis on that matter should have startled the reflective scientist; it startled the LYM team. Thinking, hours later than the discussion which that question had first provoked, I was delighted! At the next opportunity to present my case, on the following morning, I presented the team my thoughts in explanation of Gauss’s remarks. I also presented them with a footnote I had prepared the previous evening for intended publication in a major paper of mine in progress of completion at that time. This bears on a crucial feature of Vernadsky’s *On the States of Physical Space*. (Footnote: See Section I:13 of this Vernadsky work itself, also the entirety of Section II. A provisional English translation of this 1938 Vernadsky paper was presented as part of the *Festschrift* for my 85th birthday.)

“That observation, on quadratic reciprocity, typifies, exactly, the distinction to be made between Gauss’s actual method of discovery, and the frequent manner in which he not only presented, but defended his actual discovery later. I am as gratified as a “proud papa” by what that LYM team itself has done, actually independently of my explicit direction, to that effect. Here, I go a qualitative step further.”<sup>6</sup>

This reading of Lyn’s statement should provoke at least two more questions: the first is, how does one relate astrophysical “sphericalness” to “quadratic reciprocity,” and the second is, how does one relate both astrophysical “sphericalness” and “quadratic reciprocity” to music. Lyn did not elaborate on this matter, but John Sigerson touched on it during his presentation. So, I will venture a hypothesis, which I submit to you as an epistemological challenge that Lyn alluded to, when he was in prison, and which is in applying the Lydian musical ordering to the Pythagorean Quadrivium. Lyn’s investigation of that question bears on the truth of a subtle matter of epistemology which I have several times encountered, in the past 40 years or so, which is to establish an appropriate epistemological proportionality among Geometry, Arithmetic, Music, and Astronomy. And for this to succeed, one must investigate the future. Lyn’s hint was the following:

“I warned those assembled for this mission, that they must ask themselves: *What were those hidden features, and why was Gauss committed to suppressing certain among the relevant, underlying facts about his own discoveries? What is the difference between the method Gauss employed for his discoveries, and his method of presenting the proof of that which he had achieved with such justified pride? Why is there such a difference?*”

---

<sup>6</sup> Lyndon H. LaRouche, Jr., [On Vernadsky’s Space: More on the Calculus](#), EIR, Vol. 34, No. 40, October 12, 2007, pp. 40-41.. See also, Lyndon H. LaRouche, Jr., [On Vernadsky’s Space: More on Physical Space-Time](#), EIR, Vol. 34, No. 39, October 5, 2007, pp. 32-35. I ALSO RECOMMEND THE READING OF Peter Martinson’s paper on quadratic reciprocity: <https://science.larouchepac.com/gauss/ceres/InterimII/Arithmetic/Reciprocity/Reciprocity.html>

“The source of the problem lay not in Gauss himself, but in the state of mind of most among the audience to which virtually all of his discoveries were presented for publication in those times.”<sup>7</sup>

I venture to say, here, that the difference between the “method of discovery” and the “method of presenting the proof” is like the difference between Ptolemy and Kepler in dealing with the *Equant*; that is, it is located in a process which is based on a false center, or a missing center as Cusa had already established, which also begs the question of investigating the caustic region of an *axiomatic change*.

Our present times and mankind are in a similar predicament. The whole of human behavior must change to such an extent that you are as if you no longer have a leg to stand on. For instance, the crisis of the coronavirus is a case in point. There is a before COVID-19 and there is going to be an after COVID-19. What takes place in between the two is what is axiomatically important. How are you going to save mankind in the meantime? You cannot rely on what is available in order to figure out what needs to be done; that won't work. You need to figure out what are the required new ideas needed to save mankind, and then, do what needs to be done to discover those new ideas and their implementation in order to accomplish that change.

### **THINK IN TERMS OF RECIPROCITY: CORRECT YOURSELF AS YOU GO ALONG**

“Progress does not ‘use up’ progress; rather, it feeds it.”

Lyndon LaRouche, *A Deadline In Destiny*,  
EIR, June 17, 2011, p. 19.

Lyn once made a remark that Kepler recognized his own errors and corrected them as he went along. The point Lyn was making is that the universe of the scientist is not the universe of sense perception, but is rather a universe in which the individual who is willing and able to measure the way his mind works, discovers the universe with a series of discoveries of principle through reciprocity. Lyn wrote:

---

<sup>7</sup> Lyndon LaRouche, [On Vernadsky's Space: More on Physical Space-Time](#), p. 32.

“So, in the case of Kepler, you have the clearest demonstration on a large scale, of a great scientific mind, understanding the universe better, by examining its own effort to understand the universe. So, Kepler is reciprocal: Kepler presents you science, as the study of *the behavior of mind, which is making scientific discoveries; and the process of correction that [it] involves*. That’s unique.

“Then we came to Gauss. Now, Gauss is fun, because Gauss never tells the truth. That is, in very few cases, does Gauss actually present the method by which the discovery was made. Now Gauss tells the truth about one thing: When he comes up to a discovery, to present the resulting discovery, he then gives you a presentation of the way in which this discovery can be validated. Usually mathematically. But he doesn’t tell you the truth—and there’s a very good reason for it, which is relevant to what we’re doing here, today, and in society.

“The reason he doesn’t tell you the truth is because there’s a reign of terror going on. Gauss had destroyed the credibility of Euler and Lagrange. Lagrange went on to Paris, where he became a protégé of Napoleon Bonaparte, in 1799. And Bonaparte took the first step toward breaking up the Ecole Polytechnique, which was the leading scientific institution of Europe at that time, which had been formed on the brink of the French Revolution, but actually had a longer basis in the work of Gaspard Monge.”<sup>8</sup>

What is the issue, here? The issue is human creativity. Lyn makes the point that European scientists, at the time of the French Revolution, and most effectively under Napoleon Bonaparte, were targeted on the matter of truth and of the creative process; that is on the difference between man and animal.

### “WHAT IS CREATIVITY?”

“What’s the difference between man and an animal?”

---

<sup>8</sup> Lyndon H. LaRouche, Jr., [\*The LaRouche Youth Movement : International Strategy To Build a Bridge to the Future\*](#), EIR, October 19, 2007, p. 67.

“Is there a biological difference between man and a beast? One that you can determine by medical science, in the normal sense, today? No. There is none.

“What’s the difference?

“The difference is, the animal aspect of man is mortal, and dies. The human aspect of man is not mortal, and does not die. The human aspect of man, or the human individual, is not located within the confines of an animalistic body. Even though we do have an animalistic body; that’s an appendage of us!

“What the human being can do, that no animal can do, is make a fundamental discovery of universal principle, a true principle of the universe: Only a human being can do that. And it’s only through that power, the power of the human being as distinct from the animal, from any kind of animal; or for any kind of study of biology, as known today, except the effects of some of the biology, like the power of the human creative will in sometimes controlling the way the human biology functions. The difference is, that mankind, unlike any animal species, can make a discovery, and apply that discovery, which will increase the potential relative population density of the human species, or of the particular society.

“This is the power which is called ‘creativity.’ This is the power which is the *enemy* of the Second Law of Thermodynamics, so-called. Because, if you believe that the universe is organized in a way which deals with some universal law of entropy, or a fixed system, you don’t understand the universe, and you don’t understand the human mind.”

“*What is creativity?* Well, by creativity, we mean, essentially, the discovery of a universal physical principle, as typified by Kepler’s discovery of gravitation, especially in the *Harmonies*. The issue is already there, clearly, in the question of the orbit of Earth. But it is not *forced* upon you, until you face the *Harmonies*. Because, how is gravitation organized? It’s organized as Bach would have wished! The principle of gravitation is a

principle of the universe, which the fakers call the “Third Law.” But it’s not called the Third Law by Kepler. It’s what the British came along with as an explanation, to try to explain it out of the way. *It’s the power of the individual human mind to discover a principle of the universe, such that that principle, as understood by the human mind, can be employed by human beings to change the universe!*

“That’s the difference between man and the animal!”

“That’s why I had to get people into The Basement, away from the Boomers. Because the Boomer culture is rather soft on Liberalism, at least as a philosophical system, and saying, “Well, you have to be Liberal” or something. “You have to submit to this.”

“But if you want to be a scientist, you can’t be a Liberal! If you’re trying to be a scientist and you’re a Liberal, you’re wasting your time; or, you wasting somebody else’s time and money.

“The discovery of universal physical principles occurs in a universe *which is anti-entropic*, in principle. And *only the human mind, among all known living creatures, can do that.*

“That’s the difference between being an animal, and living like a beast! *All ancient history is predominantly evil*, in the sense, not that it lacks competent people, or leading people, or leading institutions. But the fact that it condemns the majority of humanity to a bestial existence, precisely as the great Greek tragedian Aeschylus portrays the fight of Prometheus in *Prometheus Bound*: People are not supposed to be allowed to discover universal physical principles, by which man is able to change the universe and man’s destiny. Human beings are supposed to behave like the cow that is well cared for, and goes into the barn, and is well treated ... until the day it’s slaughtered. That’s the Physiocratic principle, the same thing. The fundamental principle of all British economics and Cartesian systems is the same thing: *the denial of the existence of the powers of creativity, the denial of the existence of actual universal physical principles.* That’s it!

“My concern is to liberate man from slavery. And the worst slavery is not the slavery of the shackles on your hand, *it's the slavery of the shackles on your mind!*

“And you have to appreciate the fact that there is something, that you don't get taught in schools, these days; you don't get taught in textbooks, and you can leave universities quite successfully without knowing anything about it: the meaning of creativity and the meaning of anti-entropy.

“And therefore, the only way you can teach this, is, you can't teach it with a whip; and you can't teach it at a blackboard: People have to discover it and experience it, in themselves. What you have to do, is know what the mission is, and try to create the circumstances and structure the challenge, on which it is likely, that people facing that challenge, in cooperation, will interact among themselves, and will actually make, what was for them, *an original discovery of a universal physical principle.*

“That's what happened in the case of the work on the Kepler, the two phases. It became conspicuously clear in the work on the second part, on the question of the harmonics. Because, mathematics, as taught and believed by most people, does not work in dealing with universal physical principles—*it does not work.* And the *Harmonies* demonstrates it”.<sup>9</sup>

Lyn identifies creativity with the meaning of anti-entropy. The power of the creative process is the power of the truth; that is, the ability to act and think against entropy, to be anti-entropic is to fight against the second law of thermodynamics.

This may not be obvious, but this double negative “anti-entropic” truth is the essence of reciprocity in the process of self-correction, the fundamental principle of Kepler and Gauss in their astronomy works; and that can be best discovered by the observation of the orbit of the Earth around the Sun. The scientific evidence cannot be given to the eyes and to the ears; what you see and hear deceive you and must be corrected. Sense perception does not give the truth of what exists, it cannot

---

<sup>9</sup> I Lyndon H. LaRouche, Jr., [\*The LaRouche Youth Movement : International Strategy To Build a Bridge to the Future\*](#), EIR, October 19, 2007, pp. 69-70.

give the knowledge of the universal principles; it merely provide a trace of reality, a dark shadow which masks reality. This is where the mask of Gauss comes in; how can the truth of the power of discovery, which is given to man at the moment of creation, be realized so that he can “have dominion over nature?”

This can only be discovered if man looks behind the mask of things, behind what is mere appearance, mere opinion, and the first scientific task is to look at the intentions behind those who are the opinion makers, behind the intentions of those who wish you to be a slave of what you perceive. Kepler discovered his harmonies when he discovered *the power of making this discovery of principle which changes the universe, and which gives you the ability to correct your actions at the same time*. That is an awesome performative power indeed, and that is the power that Gauss was unable to wield openly, because he feared he might be killed if he did.

The wonderful insight of Lyn, here, is that “Gauss is a reflection of the creative process which wears a mask, in order to protect itself from being identified as a dangerous species.”<sup>10</sup> The mechanism Gauss used was to transform something ugly into something beautiful; he turned the ugliness of mathematics of his day into a potential for a constructive geometry of tomorrow.<sup>11</sup> The point is that you have to have a child’s mind to discover this stuff. Lyn established such a distinction when he identified the crucial reciprocal nature of the sovereign individual mind as the missing link. Lyn wrote:

“The essential issue so posed concerns the essential, axiomatic distinction of man from a hypothetical talking beast. The essential, functional form of principled distinction of man from beast, lies in the creative potential powers of each sovereign human individual mind, as the uniqueness of the discovery of universal gravitation by Johannes Kepler, or the earlier doubling of the cube by construction by the Pythagorean friend of

---

<sup>10</sup> Ibidem, p. 71.

<sup>11</sup> Zeke Boyd sent me a fascinating article on constructive geometry from Tom M. Apostol, [A VISUAL APPROACH TO CALCULUS PROBLEMS](#), Engineering & Science, No. 3, 2000. Apostol wrote: “Calculus is a beautiful subject with a host of dazzling applications. As a teacher of calculus for more than 50 years and as an author of a couple of textbooks on the subject, I was stunned to learn that many standard problems in calculus can be easily solved by an innovative visual approach that makes no use of formulas.” p. 23.



Plato, Archytas, illustrate the distinction of creative mentation from mere learning. [...]

“It is essential, that this progress within national cultures be efficiently interactive among them. Not only must such benefits of knowledge be made available to the benefit of others, but the progress of mankind as a whole depends upon promoting the raising of the level of competence of the other through promoting the sharing of these achievements in culture with one another, to the intended advantage of the other.”<sup>12</sup>

## 2. A CONSTRUCTIVE GEOMETRICAL WAY OF DIFFERENTIATING ANALOG FROM DIGITAL

*“The unexamined life is not worth living.”*

Socrates, *Apology*, (38a4-5)

The matter of more deeply understanding the ordering principle of Brahms' 4th Symphony with the opening four notes, **B-G** and **E-C**, and his application of the same reciprocals to the opening measure of the third of his *Four Serious Songs* requires an experimentation of Lyn's understanding of the Leibnizian notion of *infinitesimal* as being analog instead of digital within a characteristic process of analysis situs.

The need for reciprocity among the different phenomena of the Pythagorean Quadrivium, for example, as in Geometry, Arithmetic, Music, and Astronomy, stems from the “*analog*” nature of the higher principle underlying all universal principles, but most importantly, the universal principle of reciprocity, as it is reflected, for example, in the solution to the Delian paradox of doubling the cube by Archytas. This ontological quadratic quality is also fundamentally connected to Leibniz's notion of the *infinitesimal*. This is also what Lyn called the “Basement” method for the LYM, as he developed this matter of principle in the following terms:

---

<sup>12</sup> Lyndon H. LaRouche, Jr., [Die Euro Lüge: Capitalism & Its Law](#), EIR, Vol. 35, No. 1, January 4, 2008, p. 13.

“The issue of the calculus, as predefined by Cusa, Kepler, Fermat, and by Leibniz himself, is that a universal physical principle not only bounds the observed events of the universe, but that this occurs in a fashion which implicitly defines the thus-bounded universe as finite, as Albert Einstein was to emphasize this implication of Riemannian physics. *No part of the action which is effected within the bounds of the subject of that principle contains, formally, in digital-mathematical terms, the principle which causes it.*

*“No (digital) formal-mathematical description of the trajectory of a principled form of action, such as gravitation, contains the principle itself within it. It is only the replication of the experience of the relevant crucial experiment itself, which proves the validity of a claim for a universal physical principle. [Footnote: This is known to the LYM teams as the “Basement” method.] Hence the requirement for “analog methods.”*

*“Therefore, whereas, the effect of the principle’s action is clearly manifest empirically, the principle itself (e.g., gravitation) is not confined by the subordinated domain (the orbital pathway) upon which it acts. Therefore, as Sky Shields presents the case, the apparently ontological connection of the principle to the subsumed action, can only be estimated as a point of contact which is ontologically (not spatially) infinitesimal at each and all point-intervals which might be adopted.*

*“Ontologically, the principle controls the action, but the action does not control, and does not contain the principle at that, or any other point: hence, gravitation as such is expressed as an absolutely (e.g., inherently) infinitesimal mode of action.*

“That is the underlying principle of the work of such followers of Cusa and Kepler, as Leibniz and Riemann (for example). It is also, methodologically, the key to the genius of Academician V.I. Vernadsky and Albert Einstein.

*“Hence, on the basis of such evidence, as Shields illustrates this point, we must go a step further, to say, that no derivatives of digital methods*

*could ever encompass the action which corresponds to a true universal physical principle.*

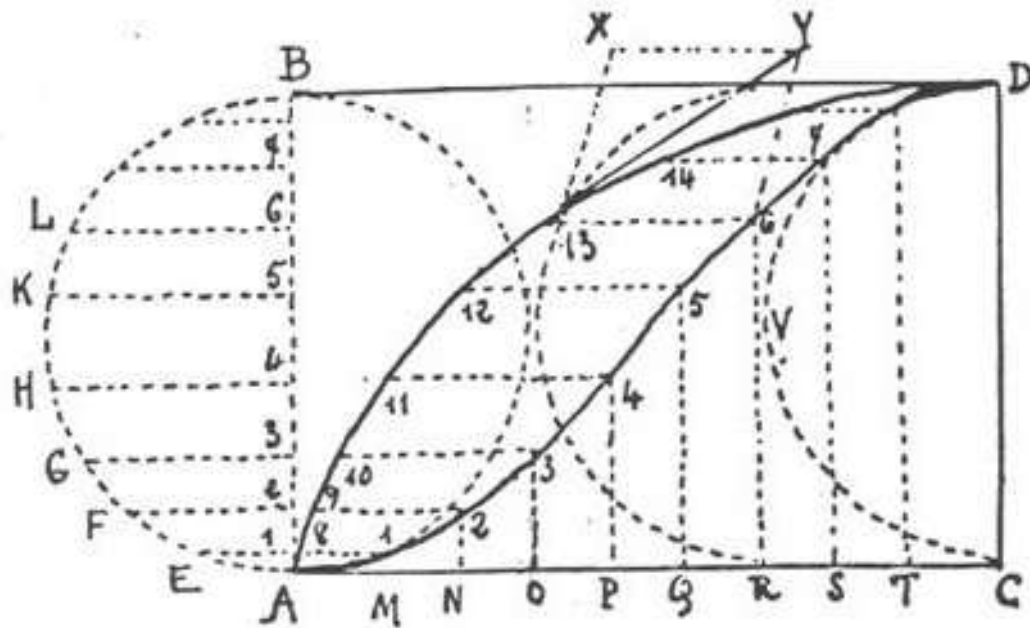
“That is the one and only actual meaning of the use of the term *infinitesimal* by Leibniz. Euler knew this fact from Jean Bernouilli’s published, well-documented representations of the work of himself and Leibniz. Furthermore, the notion of least-action itself, as proposed by Leibniz, reflected, as by Leibniz’s explicit emphasis, the notion of least action introduced by Fermat, as that fact was also well known to Euler. Furthermore, the notion of both the calculus itself, and of the challenge of physical-elliptical functions, had been proposed to future mathematicians by Kepler; there was never any margin of opportunity for a competent scientist of the Seventeenth or Eighteenth centuries, taking these matters of background into account, to make an “honest” mistake in respect to the ontological content of the subjects to which such terms had referred.”<sup>13</sup>

I wish to conclude this part with a matter of constructive geometry that Zeke Boyd brought to my attention recently which demonstrates very beautifully how constructive geometry without the formalities of the calculus is able to solve the opposition between digital and analog. The proof of the matter lies in the discovery of the cycloid by Gilles Personne de Roberval (1734) and of a discovery by a present day little known Armenian-American scientist by the name of Mamikon A. Mnatsakanian who has been using a similar method of constructive geometry.<sup>14</sup>

---

<sup>13</sup> Lyndon H. LaRouche, Jr., [Life Within the Noosphere: What Is the Human Mind?](#) EIR, Vol. 35, No. 2, January 11, 2008, PP. 49-50

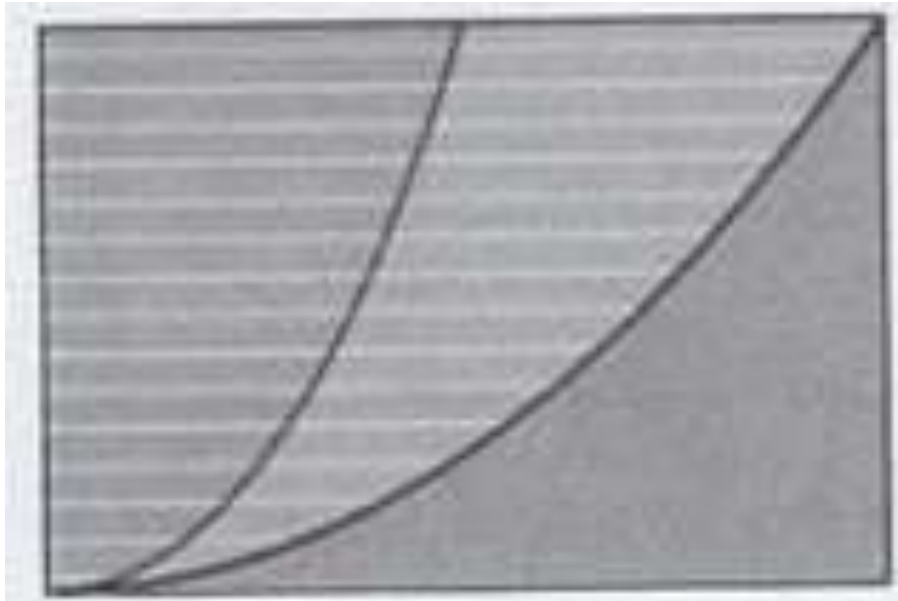
<sup>14</sup> See Tom M. Apostol, [A VISUAL APPROACH TO CALCULUS PROBLEMS](#), Engineering Science No. 3. 2000. pp. 23-31.



Roberval's cycloid

What Mnatsakanian rediscovered, in his own right, is the geometrical method of Roberval, and most emphatically, his method of constructing “indivisibles” (infinitesimals) as reflected in Roberval’s original construction of the cycloid. After discovering that the area between the cycloid curve and the sine curve was equal to the area of the half circle, Roberval found that the total area under the cycloid curve was precisely three times the area of the generating circle.

The method is so simple that it can be easily understood by a child with an appropriate guiding hand. The key is to be able to do the construction without the use of the calculus. This is what Mnatsakanian did, when he constructed the area under the parabola by using the principle of reciprocity for determining the area under the cycloid.

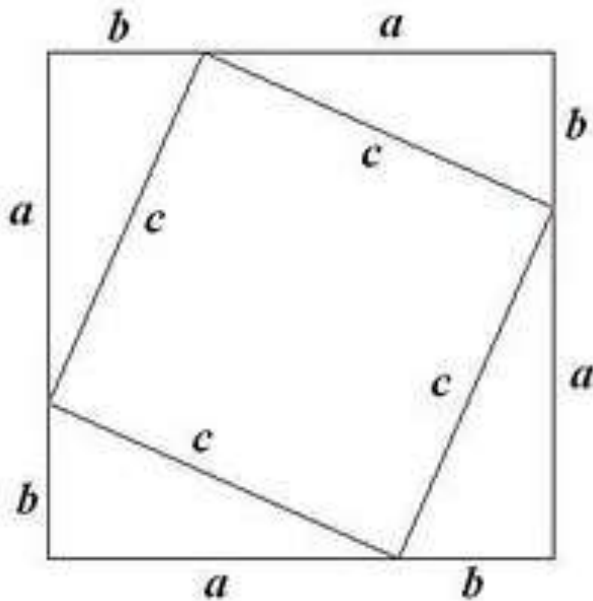


Mnatsakanian's parabola. <sup>15</sup>

In order to find the area under the parabolic segment (left rectangle), which covers less than half of the total area of the rectangle, Mnatsakanian divided the area above the parabolic segment into two less shaded equal areas, each of which enclosed equal amounts of “indivisibles,” as in Roberval's cycloid. For the same reason, since those two areas corresponded to two thirds of the whole rectangle, the area under the parabolic segment (the Pythagorean gap) had to be equal to  $1/3$  of the said rectangle! This method is very similar to the Pythagorean method of discovering the double of the square by means of what is not there.

---

<sup>15</sup> Op. Cit, p. 28.



Big square:  $(a + b)^2$   
 $= a^2 + 2ab + b^2$

Four triangles =  $2ab$

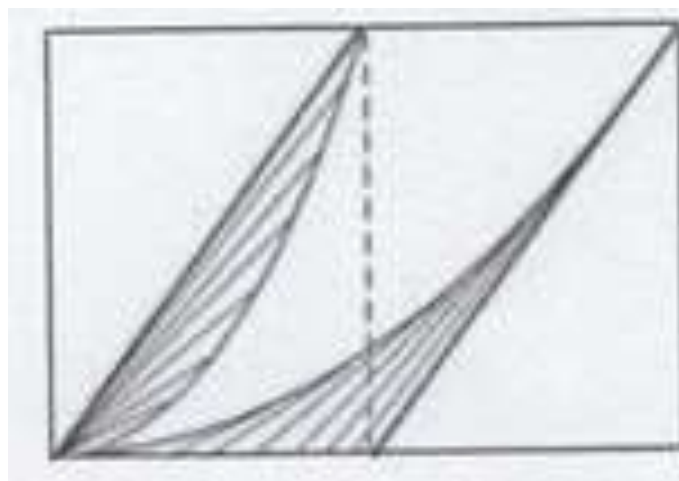
Little square =  $c^2$

$a^2 + 2ab + b^2 = c^2 + 2ab$

$a^2 + b^2 = c^2$

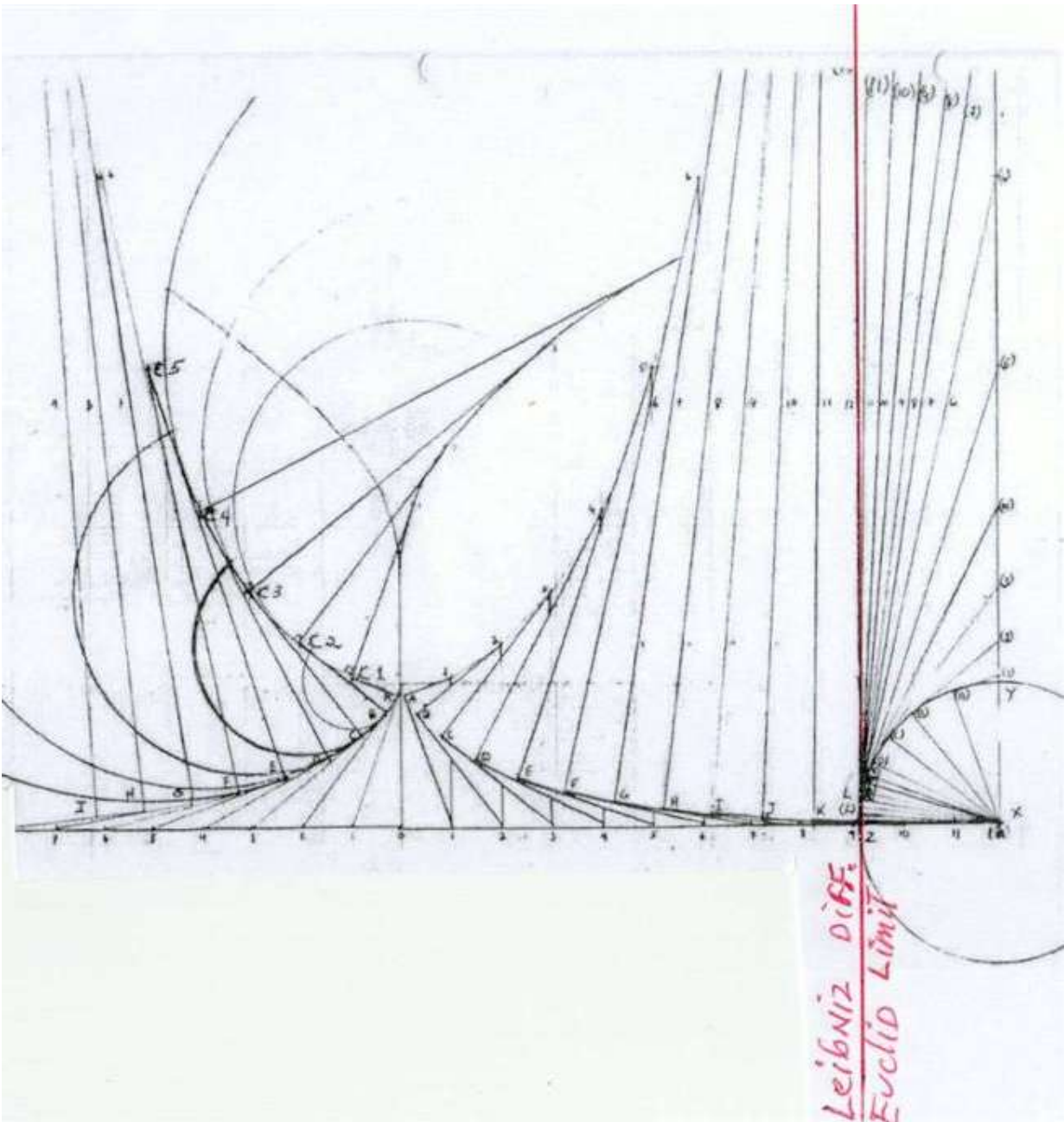
Pythagorean method for doubling the square.<sup>16</sup>

Next, Mnatsakanian found that the area of the parabolic sweep was equal to the area of the parabolic cluster (right rectangle). His theorem is: *“The area of a tangent Sweep is equal to the area of its tangent cluster, regardless of the shape of the original curve.”*<sup>17</sup>



<sup>16</sup> The Pythagorean Theorem was subverted very early on by the Oracle of Delphi and was reduced to a mere algebraic exercise of finding the third side in a right angle triangle.

<sup>17</sup> Op. Cit, p. 26.



Catenary-tractrix construction by Pierre Beaudry.

This Leibnizian method of construction of the catenary-tractrix is the key to my own discovery that the area of the tangent cluster above the generating quarter circle (on the right of the axiomatic red line) is equivalent to the area of the right half of the tangent sweep of the catenary curve (on the left of the same axiomatic

red line). For the same reason, the area of the radius cluster of the quarter circle is also equal to the area under the right half of the tractrix curve.

### 3. KEPLER'S DISCOVERY OF THE HARMONIES AND THE LYDIAN QUADRATICS

One of the most precious discoveries in all of human history was Kepler's discovery of the musical harmonies of the Solar system. In *The Harmony of the World*, Kepler established a very convincing hypothesis whereby there can only be Five Constructible Platonic Solids, because the harmony of the five planets observable during his time reflected the harmonies of the Octaves of the Musical System. This hypothesis also holds true for what holds together the gravitational analog of the Pythagorean Quadrivium. Kepler's hypothesis is very simple:

“Come now, let us see whether what we have already inferred by reasoning is in actual fact found to be so. However, let us preface this with some words of caution, to avoid being obstructed while the inquiry is in progress. First, we should for the present overlook those excesses, or deficiencies, which are less than a semitone; for we shall see later what causes them. Next, by repeating doubling, or on the contrary by halving, of the motions, we shall bring them all within a system of a single octave, because of the identity of sound of every diapason.

“[...] Now let the motions be compared, in parts obtained by continuous division by two.

“Then of the motion at	<i>min. sec.</i>
Perihelion of Mercury the seventh halving, or 128 <sup>th</sup>	is 3. 0.
Aphelion of Mercury the sixth halving or, 64 <sup>th</sup>	is 2. 34. -
Perihelion of Venus the fifth halving, or 32 <sup>th</sup>	is 3. 3. +
Aphelion of Venus the fifth halving, or, 32 <sup>th</sup>	is 2. 58. -
Perihelion of Earth the fifth halving, or 32 <sup>th</sup>	is 1. 55. -
Aphelion of Earth the fifth halving or, 32 <sup>th</sup>	is 1. 47. -
Perihelion of Mars the fourth halving, or 16 <sup>th</sup>	is 2. 23. -
Aphelion of Mars the third halving, or, 8 <sup>th</sup>	is 3. 17. -
Perihelion of Jupiter half	is 2. 45.
Aphelion of Jupiter half	is 2. 15.



and the motion at  
 Perihelion of Saturn  
 Aphelion of Saturn

is 2. 15.  
 is 1. 46.”<sup>18</sup>

The same harmonic ordering holds true for the quadratic Lydian interval divisions of the Well-Tempered Musical System in the following table made by my friend Bill Bohdan, except that the division is not among several octaves but within a single octave of the same series of middle C-256 to C-512.

THE PLANETARY ORBITS AND THE EQUAL-TEMPERED MUSICAL SYSTEM							
by WILLIAM BOHDAN							
PLANETS	ASTRO. UNITS	Log. 10X	ADDED CONSTANT	MULTIPLE CONSTANT	CYCLE EQUIVALENT	MUSICAL CYCLES	PLANETS
MERCURY	(P) 0.310	0.5086	+2.496	x 128.8	255.97	C = 256	MERCURY
MERCURY	(A) 0.470	0.3279	" "	" "	279.25	C# = 271.22	MERCURY
VENUS	(P) 0.715	0.1457	" "	" "	302.72	D = 287.35	VENUS
VENUS	(A) 0.725	0.1397	" "	" "	303.49	Eb = 304.44	VENUS
EARTH	(P) 0.983	0.0074	" "	" "	320.52		EARTH
EARTH	(A) 1.017	0.0073	" "	" "	322.42	E = 322.54	EARTH
MARS	(P) 1.379	0.1396	" "	" "	339.46	F = 341.72	MARS
MARS	(A) 1.661	0.2204	" "	" "	349.86		MARS
ASTEROIDS	(P) 2.2	0.3424	" "	" "	363.32	F# = 362.04	ASTEROIDS
ASTEROIDS	(A) 3.6	0.5563	" "	" "	393.13	G = 383.57	ASTEROIDS
JUPITER	(P) 4.95	0.6946	" "	" "	410.95	Ab = 406.37	JUPITER
JUPITER	(A) 5.45	0.7364	" "	" "	416.33		JUPITER
SATURN	(P) 9.006	0.9545	" "	" "	444.43	A = 430.54	SATURN
SATURN	(A) 10.074	1.0032	" "	" "	450.69	Bb = 456.14	SATURN
URANUS	(P) 18.288	1.2622	" "	" "	484.05	B = 483.26	URANUS
URANUS	(A) 20.092	1.3030	" "	" "	489.31		URANUS
NEPTUNE	(P) 29.799	1.4742	" "	" "	511.36		NEPTUNE
NEPTUNE	(A) 30.341	1.4820	" "	" "	512.37	C = 512	NEPTUNE

The ordering, here, reflects the quadratic Lydian interval divisions of the equal-tempered octave; that is, **C, E b, F#, A, C**. The secret, here, is that with these quadratic Lydian divisions, including the two other series of **C#, E, G, B b**, and **D, FD, Ab, B**, you can generate, as J.S. Bach did, any piece of musical composition in all of the twelve keys of the well-tempered musical system. Biquadratic residues show how such Lydian intervals behave inside of a torus.

<sup>18</sup> Johannes Kepler, *The Harmony of the World*, Translated into English by E.J. Alton, A.M. Duncan, and J.V. Field, American Philosophical Society, 1997, p. 432.

#### 4. BENJAMIN BANNEKER'S DISCOVERY OF THE ARITHMETICAL QUADRATIC ANALOG OF PROPORTIONALITY

Moreover, these Lydian quadratic intervals also have a fascinating relationship to the principle of ordering the different functions of addition, subtraction, multiplication, and division in simple arithmetic. I refer, here, to the famous Banneker Puzzle, which also uses a quadratic partitioning similar to the Lydian quadratic partitioning of the musical octave.<sup>19</sup> Banneker's puzzle states:

*“Divide 60 into four such parts that when the first being increased by 4, the second decreased by 4, the third multiplied by 4, the fourth part divided by 4, that the sum, the difference, the product, and the quotient shall be one and the same number.”*

The answer to Banneker's puzzle is as follows:

First part    5.6 increased by 4 = 9.6  
Second part 13.6 decreased by 4 = 9.6  
Third part    2.4 multiplied by 4 = 9.6  
Fourth part  $\frac{38.4}{60.0}$  divided by 4 = 9.6

The question is how can one find the four parts which total 60? This is where the future enters into play. The answer has to be found by a process of inversion from the future to the past; that is, from a quadratic function that must be projected into the future and proceed by time reversal. I will give you a hint: you are looking for a quadratic analogy.

Since number 4 is the only known constant number in the Banneker puzzle, and since all four arithmetic operations must have the same result, the solution to the puzzle must require some form of quadratic solution from such a combination. The analogy you are looking for is with number 4. It is the case that all principles are discovered from the end and not from the beginning. So, let's hypothesize the following: if  $0 + 4 = 4$ , then it follows that the result of each of the other three operations will proceed in a similar manner; that is, by going back to the source:

---

<sup>19</sup> See my report: [A CONTRIBUTION TO THE PYTHAGOREAN QUADRIVIUM](#).

The sum is.....  $0 + 4 = 4$   
The difference is ....  $8 - 4 = 4$   
The product is .....  $1 \times 4 = 4$   
The quotient is ....  $16 \div 4 = 4$   
Total .....  $25 \div 4 = 6.25$

By adding the sum, the difference, the product, and the quotient of the different parts of 25, you can establish the analog function of proportionality as the principle by means of which Banneker was able to make his calculation, that is, 6.25, because his total number of parts is  $60 \div 6.25 = 9.6$ . Once you have discovered the quadratic principle behind this “magic” number, 6.25, then, you have discovered that you can apply it to any other number you wish, provided it applies to the analog quadratic principle you have “*pre-established*.”

The most interesting feature of Banneker’s discovery of the quadratic principle is when you apply the Puzzle’s partitioning to octaves of 25 units, such as 100, 75, 50, and 25, they reflect the Lydian partitioning of the musical octaves of the C-256 series. It is as if the quadratic function was acting as a modulator of change in both the arithmetic and the musical domains of the Quadrivium. Do they also work in the Astronomical and geometrical domains?

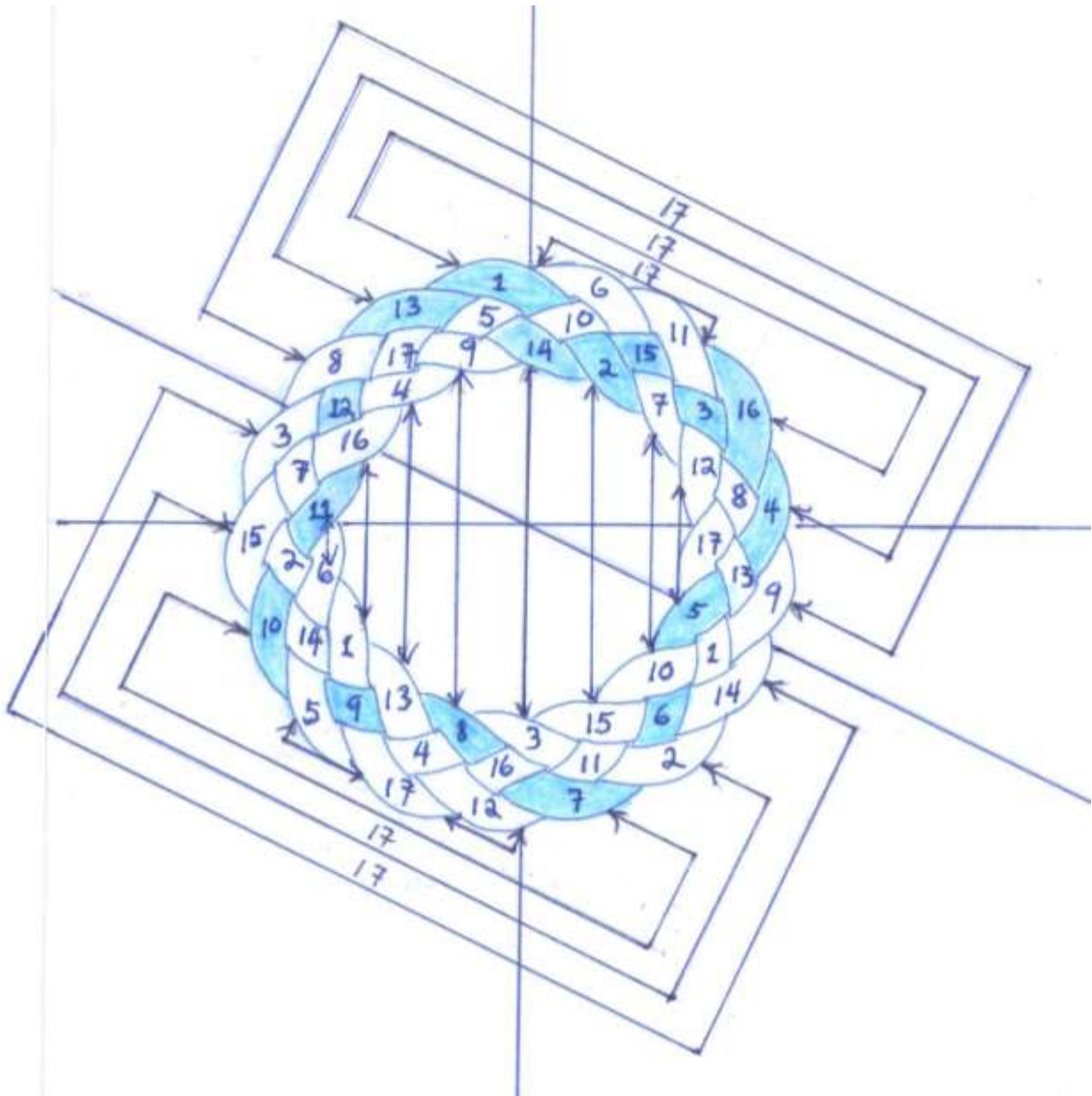
## 5. POINSOT’S DISCOVERY OF THE GEOMETRY OF PRIMITIVE ROOTS

The following last exercise will help the reader discover how constructive geometry can make sense of one of the most recondite problems in Number Theory, the question of “primitive roots” that Euler considered unsolvable.

The following figure is a modified theorem for primitive roots and biquadratic residues which has been derived from Louis Poinsot:<sup>20</sup> *If you have T wave intervals arranged in a torus, and you join them from P to P, P being a biquadratic residue of T, you will necessarily pass through all of the T intervals twice before returning to your starting point.*

---

<sup>20</sup> The general theorem can be found elaborated in my report on [FUSION POWER IS NOT DEMOCRATIC](#)



Poloidal wave 4 is a biquadratic residue of 17, as are the reciprocals (4, 16, 13, 1). Not all of the reciprocals of 17.

Using the Poincaré principle in connection with the Leonardo knotworks weaving patterns, you can easily discover how numbers are mere footprints of cycles. The way they relate to one another is through a least action form of cycles which apply to the Pythagorean Quadrivium. Let me first illustrate with Geometry

and Arithmetic how this process works. The following form of biquadratic residues, for example the case of 4 as a biquadratic residue of 17.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	16														
2	4	8	16	15	13	9	1								
3	9	10	13	5	15	11	16	14	8	7	4	12	2	6	1
4	16	13	1												
5	8	6	13	14	2	10	16	12	9	11	4	3	15	7	1
6	2	12	4	7	8	14	16	11	15	5	13	10	9	3	1
7	15	3	4	11	9	12	16	10	2	14	13	6	8	5	1
8	13	2	16	9	4	15	1								
9	13	15	16	8	4	2	1								
10	15	14	4	6	9	5	16	7	2	3	13	11	8	12	1
11	2	5	4	10	8	3	16	6	15	12	13	7	9	14	1
12	8	11	13	3	2	7	16	5	9	6	4	14	15	10	1
13	16	4	1												
14	9	7	13	12	15	6	16	3	8	10	4	5	2	11	1
15	4	9	16	2	13	8	1								
16	1														

The four biquadratics, 1, 4, 13, and 16 of number 17 are in brown and the eight primitive roots 3, 5, 6, 7, 10, 11, 12, and 14 are in pink.

## CONCLUSION

I am reminded, in conclusion, that if the quadratic proportionality of the Banneker Puzzle is an *arithmetical* modulator of change similar to the *musical* Lydian Quadratic modality in the well-tempered system of Bach, there must also be quadratic modalities in the domains of *astronomy* and *geometry* in order to complete the unity of principle of the Pythagorean Quadrivium as announced at the beginning of this paper.

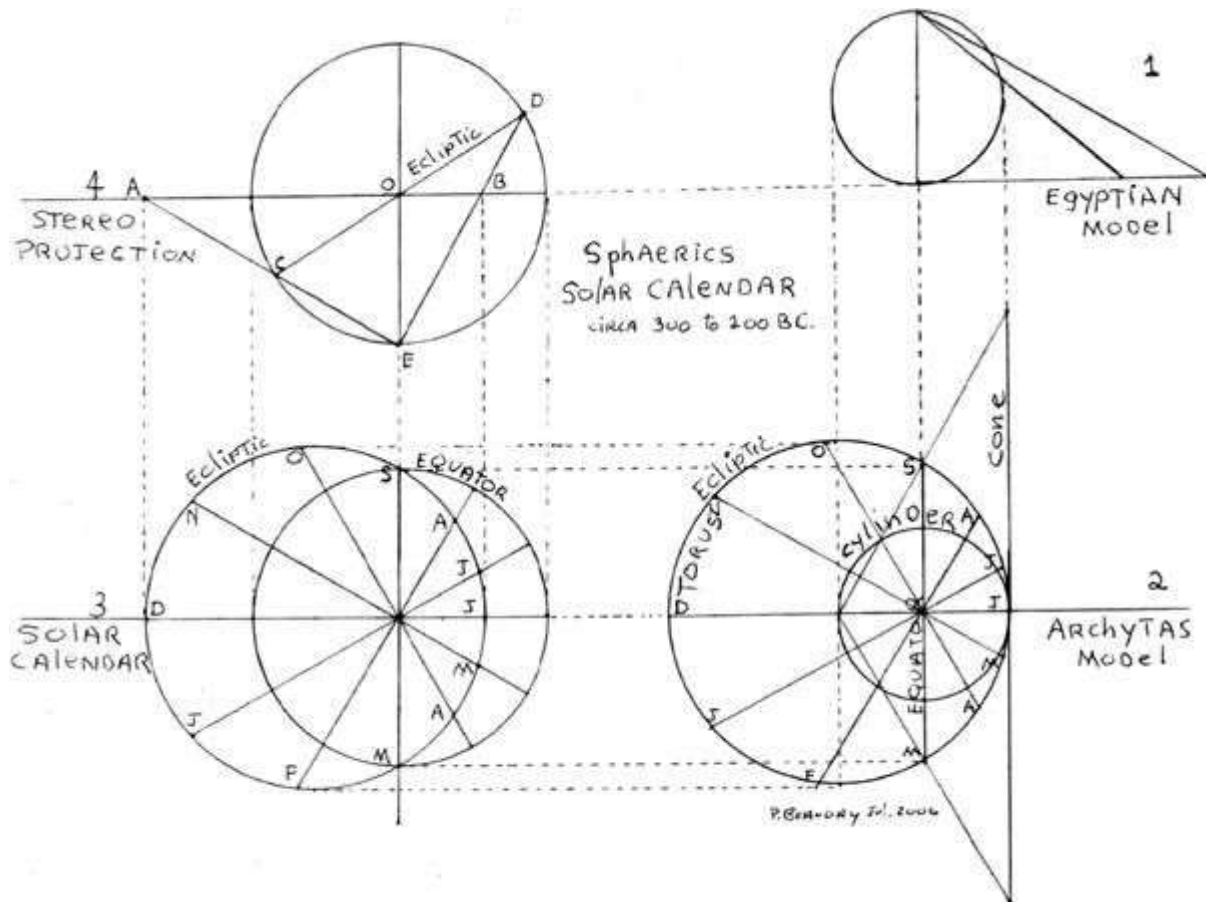
I remind the reader that the principle of quadratic reciprocity is also applicable to the construction of the Great Pyramid of Egypt whose purpose was to provide a pedagogical means of teaching astronomical Sphaerics, as it was originally devised and understood by the ancient people of the seas and as it was later expressed by a corresponding geometrical quadratic analog devised by Archytas in his famous geometrical construction for solving the Delian dilemma of the *Doubling of the Cube*.<sup>21</sup>



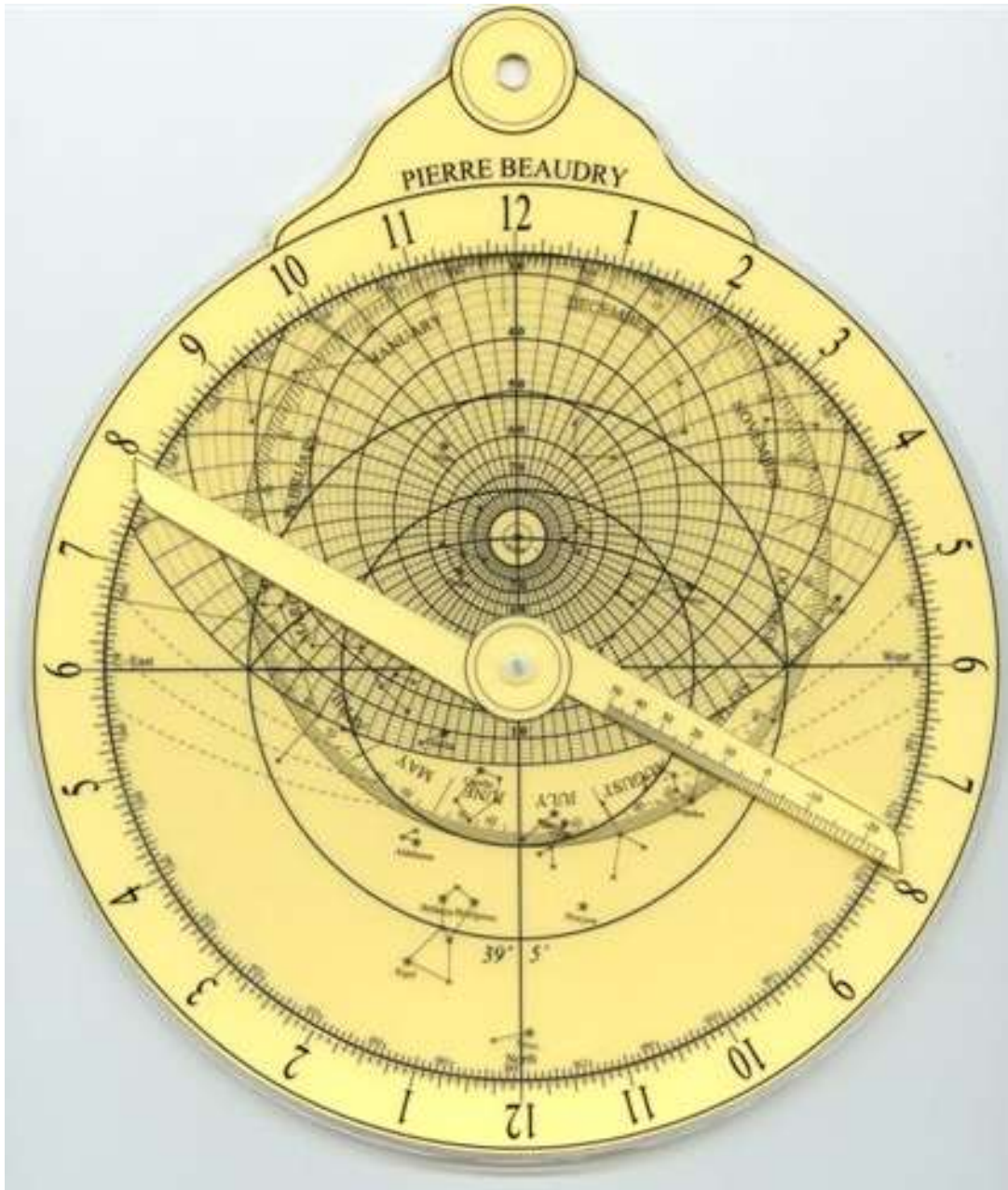
The quadratic proportionality of the Great Pyramid of Egypt corresponding to the Archytas requirement for the doubling of the cube.

<sup>21</sup> See my reports: [PYTHAGOREAN SPHERICS: THE MISSING LINK BETWEEN EGYPT AND GREECE](#); [ARCHYTAS AND THE PRINCIPLE OF PROPORTIONALITY](#); [PYRAMID OF EGYPT AND ARCHYTAS, I](#); [ARCHYTAS AT DELPHI](#); [DOUBLING THE CUBE BY MUSICAL INTERVALS](#); [GALACTIC THINKING AND THE DELIAN SOLUTION](#).

This can be illustrated briefly by the following quadratic proportionality common to both Archytas and to the Ancient Egyptian builder of the Great Pyramid of Giza.



Application of the Great Pyramid model projected against the Archytas model establishing the original solar calendar based on the Ecliptic and the Celestial Equator, both of which later giving birth to the modern geometry of the Astrolabe.



My personal Astrolabe for the Leesburg Va. region of the United States

**FIN**