
PLATO'S *EXAIPHNĒS*: MEASURING AXIOMATIC CHANGES

Thales, Pythagoras, Plato, Nicholas of Cusa, Leibniz, and LaRouche: the sudden
measuring of an axiomatic transformation

by Pierre Beaudry, 2/3/2022

FOREWORD

Plato's use of the adverbial expression "suddenly" (*exaiphnēs*) is an appropriate metaphor for identifying the transformative nature of an "instantaneous" axiomatic change inside of the human mind. Lyndon LaRouche identified this as the transfinite measure of a discovery of principle of going from a lower to a higher manifold. In the *Parmenides*, Plato qualified such a changing state of mind as a "sudden instantaneous moment" which he identified as the *unifying mental action* of a One over the Many.

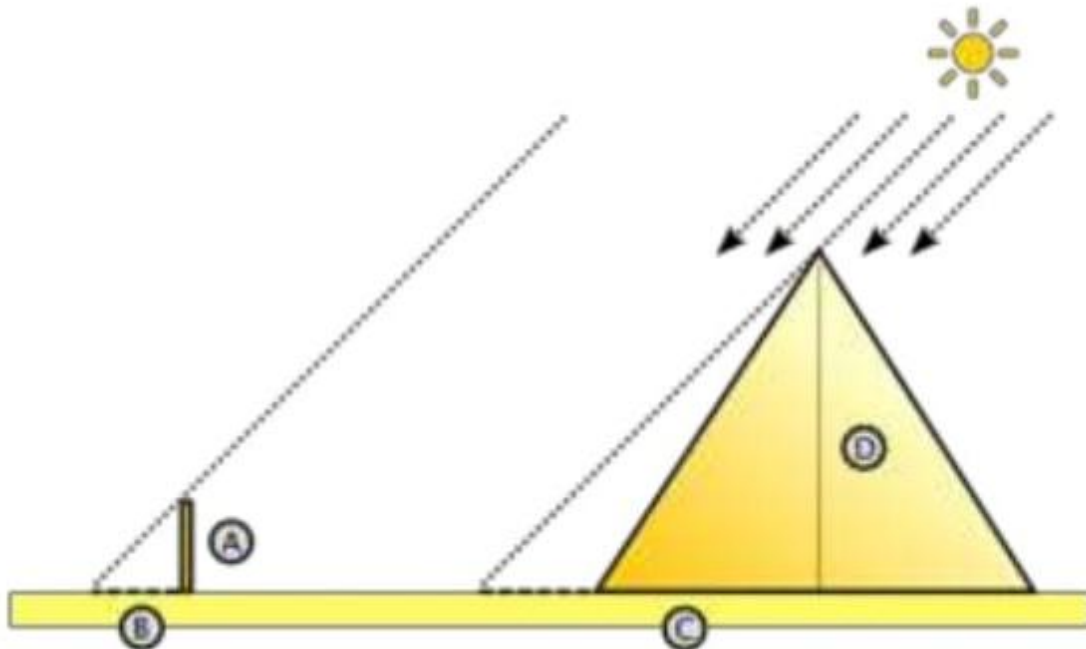
Throughout European history, the primary advocates of such an epistemological function of the human mind have been Thales, Pythagoras, Plato, Nicholas of Cusa, Gottfried Leibniz, and Lyndon LaRouche. These thinkers have used such a timely form of action for the same historical purpose, which is to modify and measure the power of the human mind with respect to God and the infinite for the common benefit and progress of mankind.

In that sense, "suddenly" (*exaiphnēs*) represents an instantaneous and unforeseen action of change which reflects the state of perplexity of the thinking person at the decisive moment of discovering, not merely the growing capacity of his or her mind at some moment in history, but also, the power of going beyond the limits of the apparent finite domain of knowledge, by measuring the critical steps

of an unbounded human transfinite progress; thus, proving by factual demonstration that there are no limits to growth.

THE EPISTEMOLOGICAL SIGNIFICANCE OF THALES' DISCOVERY OF THE HEIGHT OF THE GREAT PYRAMID

Plato's use of the idea of a sudden (*exaiphnēs*) instantaneous moment refers to a true Promethean act of discovery of the human mind's natural power to grow and progress. One of the first experiments that was used to measure such a progress of the human mind in ancient Greece came from Thales of Miletus (c. 624 – c. 548 BC) and his hypothesis for discovering a most creative method of measuring the height of the Great Pyramid. This experiment is delightful in that it involves five different entities which, when related together appropriately, have the power not only to make discoveries beyond sense perception, but most importantly, to also establish that the human mind must become part of the measuring process itself.



Thales of Miletus measurement of the Great Pyramid. From [Math story #2 - Thales and the Great Pyramid of Cheops](#)

If you connect together the five following entities, as Thales did, you will not only be able to discover the precise height of the Great Pyramid, but you will also discover the real power that your own mind has over sense perception by measuring everything you know, provided you include yourself into the measuring process. Those five components are: 1) the Sun, 2) the pyramid, 3) the shadow of the pyramid, 4) your own shadow, and 5) your mind's ordering of the previous four elements. How can those five different components come together and help you make a discovery of principle? Take the time to examine the situation thoroughly before you look for an answer.¹

First of all, imagine yourself standing next to the Great Pyramid of Egypt. Take a stick which corresponds to your height **A** and mark the length of the Sun's shadow of that height as **B** on the ground at the same time that the Sun strikes the Great Pyramid. What is the epistemological value of the idea of comparing a known to an unknown? The lesson, here, is that in all of man's investigations, the one that is the most illuminating is the one which includes the mind of the observer into the process of discovering the relationship between the known and the unknown.

The idea behind this theorem, therefore, has a profound epistemological implication. Thales was the first Greek philosopher to discover that by putting himself into the equation, and especially by including his own mind in discovering the unknown, man becomes the measure of the universe. In other words, Thales used himself as the measure of the known (his own height) in order to discover the unknown (the Pyramid's height). All that he was required to do was to discover the precise length of his own shadow.

When the length of your own shadow to your own height is the same as the length of the pyramid's shadow to its own height, then the discovery of the height of the Pyramid becomes known simply by a proportionality of triangles, in which $B/A = C/D$. However, the unit of measure that Thales wanted us to discover was

¹ The reader can find an extended discussion of this problem in my report: [THE THALES THEOREM AND THE ARCHYTAS MODEL](#).

not simply the height of the Pyramid, but, also, the fact that human thinking is the true measure of progress in the universe. Thus, the most profound discovery of Thales is the discovery of the human ability to discover his own measuring power.

THE PYTHAGOREAN THEOREM: DISCOVERING THE POWER OF DISCOVERY

Following in the footsteps of Thales, the actual discovery of the Pythagorean Theorem is not only the discovery of the third side of a triangle, or the means of discovering the area of a third square; it is more fundamentally a way to discover how the creative process itself works by discovering the future of *what is not yet there, but is about to come into existence*; that is, by discovering the power that you didn't know you had, the domain of what Lyndon LaRouche called the *higher hypothesis* and the *hypothesizing of the higher hypothesis*.

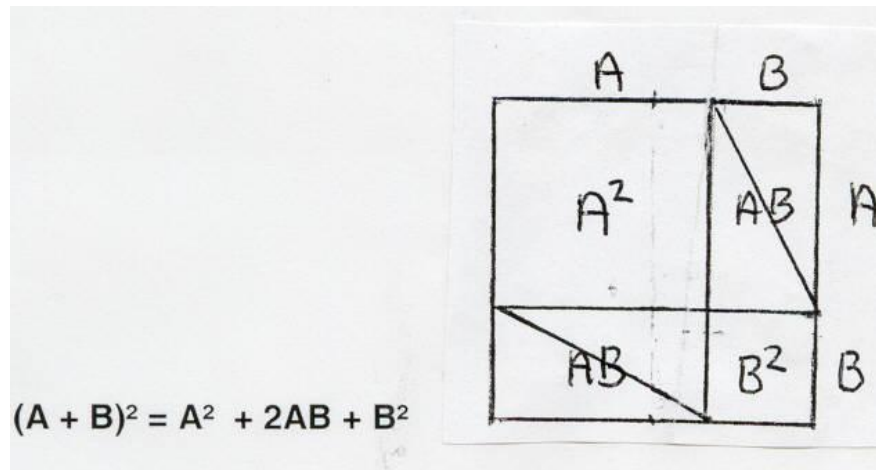
What Pythagoras succeeded in doing with his theorem is to go beyond the deductive method of logical thinking by changing the boundary conditions of your mind in such a way that you are best disposed to make any sort of discovery whatsoever. As LaRouche said:

“Those non-deductive solutions, solutions by methods which cannot be represented explicitly by any linear medium, such as communications media, typify the class of thought-objects to which belong the pupil's reliving of Pythagoras' discovery and of Cusa's discovery of an isoperimetric species of circular action absolutely distinct from the species of all possible linear functions.”²

Once you have solved the problem of doubling the square, as Plato demonstrated it in his *Meno* dialogue, you are ready to discover that the Pythagorean Theorem is derived directly from a similar process. Did Socrates not say very clearly that learning is recollecting, and that “teaching is simply being reminded” (*Meno*, 82b8)? Ask yourself then: How can I geometrically construct the principle of composition whereby $A^2 + B^2 = C^2$? In other words, given only

² Lyndon LaRouche, [On the Subject of Metaphor](#), Schiller Institute, Part I of II, from Fidelio Magazine, Vol. 1. No 3, Fall 1992.

two square figures such as A^2 and B^2 , how can you find C^2 ? How can a change in that geometrical arrangement of the following figure lead you to *what is not yet there*?



The puzzle of finding the missing squared area, C 2

At this point, as in the problem of doubling the square in the *Meno* dialogue, the reader becomes perplexed (*Meno*, (84c6). Where is C^2 in the above figure? It is not there. How can that puzzle lead you to discover the principle of creativity that will make you discover C^2 ?³

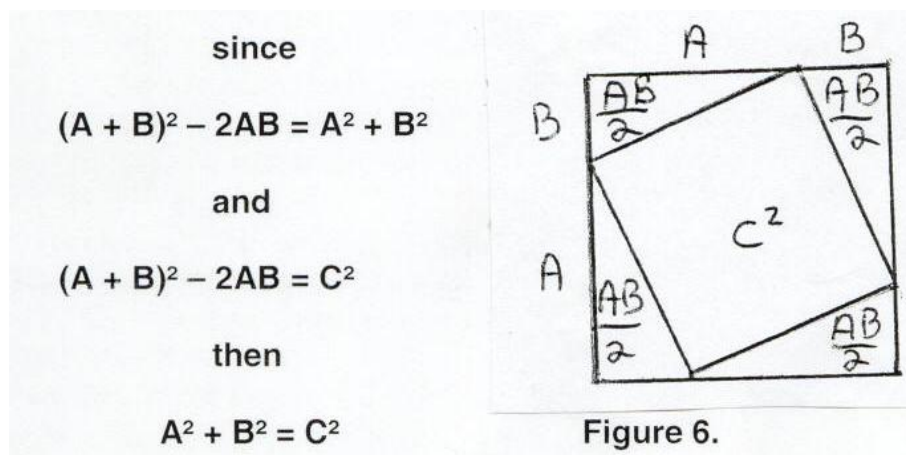
The idea to focus on is that of transforming the internal boundary conditions of your state of mind by making some sudden changes; that is to say, by changing the axioms which control what you think and prevent you from discovering C^2 . You don't need to know what those axioms are; all you need to do is to remember that you have to change things around inside of your mind. Consider that if the mind is required to change, it must change suddenly in the manner in which Heraclitus understood change; that is, within the mind in which "everything must change, except change itself." So, the question is: how can you change without change? Note that $(A + B)^2$ does not change, but everything else inside of that square can and must change.

³ The following two pages are a corrected version taken from a previous report: [ON THE CONSTRUCTIVE GEOMETRY OF PYTHAGORAS, NICHOLAS OF CUSA AND PLASMA PHYSICS.](#)

This appears to be a contradictory deductive notion because it seems to imply that everything changes and remains the same at the same time. Deductive logic is not happy with this sort of thing, because logic cannot accept that something and its opposite exist at the same time. So, something has to go: either *deductive logic* or the *unity of opposites*. My choice is to drop the former and keep the latter, because with the former, nothing ever changes. Heraclitus reminds us that like the waters of a river, everything flows and changes, but the river stays the same.

However, there is, here, a general misunderstanding where most historians think that the point Heraclitus is making is simply that everything changes period. That's not true. The point, as Plato showed, is that *the One changes the Many*, and that is what is really perplexing.

The question is: "*How do you discover the principle which causes everything to change the past in your own mind?*" The answer is that you look for *what is not yet there*, or for the inverse of what is already there. In other words, you have to get rid of the old axioms that prevent you from going to a higher hypothesis. So, you look for C^2 by changing the internal boundary conditions of the previous figure without changing the external boundaries themselves. That is how the memory function works with the help of a higher hypothesis.



The discovery of *what is not there*; that is, C^2

If you have followed this process of change closely, your mind has likely gone through three successive states: 1) Perplexity, 2) Awe, and 3) Laughter. Here is how the discovery of *what is not yet there* can best be expressed in a sudden moment of triumph:⁴

“For twenty years, Mr. Sokoloff had been eating at the same restaurant on Second Avenue. On this night, as on every other, Mr. Sokoloff ordered chicken soup. The waiter set it down and started off. Mr. Sokoloff called, ‘Waiter!’

‘Yeah?’

‘Please taste this soup.’

The waiter said, ‘Hanh! Twenty years you’ve been eating the chicken soup here, no? Have you ever had a bad plate?’

‘Waiter,’ said Sokoloff firmly, ‘taste the soup.’

‘Sokoloff, what’s the matter with you?’

‘Taste the soup!’

‘All right, all right,’ grimaced the waiter. ‘I’ll taste –where’s the spoon?’

‘Aha!’ cried Sokoloff.”⁵

PLATO’S SUDDEN (*exaiphnēs* - ἐξαίφνης) DISCOVERY OF THE PRINCIPLE OF THE ONE OVER THE MANY

How did Plato *hypothesize the higher hypothesis* of the principle of the One over the Many? The most important measuring rod of knowledge that Plato introduced in his corpus of philosophical education for the benefit of the philosopher king of the Greek City-State, can be found in his best known and most significant Platonic story: the allegory of the cave [*Republic*, Book VII (514a-521b)]. If you pay close attention to the profound significance of this story, you

⁴ Michelle and Poul Rasmussen sent me a geometrical poem by Hans Christian Anderson on how to prove the Pythagorean Theorem: <https://www.gathering4gardner.org/g4g13gift/art/NashJane-GiftExchange-EternalMagic-G4G13.pdf>. A Chinese proof of the Pythagorean Theorem can also be found online: https://external-preview.redd.it/K9Ou85QmfW3HEoO-R2rXgXEm3n5f_iV12uYxcOxD7ng.jpg?auto=webp&s=836ae696c55c8108c1a2d2b5ccf746e57dd92a7f

⁵ Leo Rosten, *The Joys of Yiddish*, Pocket Books/Washington Square Press, 1968, p. 6.

will also discover the epistemological measure of the “sudden” (*εξαίφνης* - *ἐξαιφνης*) discovery of an axiomatic change which transforms the human mind by having it go from a lower to a higher manifold. *The nature of the change is similar to that of an adverbial modification of a verbal action in the simultaneity of eternity.*

The allegory of the cave begins with Socrates making us discover the nature of our wrongly manipulated education; that is, where human beings are being held captive as prisoners in a cave which is open to the outside light of truth behind them, which they are not allowed to see or to access. They are unaware of the fallacy of the condition they have been put into, and reality for them is nothing but an illusion projected on the dim wall of that cave. Plato's allegory is meant to illustrate the fact that the whole of society is actually corrupt and based on a false system of education which fabricates for its citizens nothing but lies and shadows of the real world. The prisoners' lives are spent watching mere illusions projected on the TV screen of their cave by hidden manipulators who operate behind their backs and make them believe they are free democratic citizens.

Socrates asks Glaucon to try and imagine what would happen to such prisoners if they were to be liberated from their illusions; that is, if they were to break their chains and turn around to walk back toward the light of truth which is located behind them outside of the cave. Socrates answers as follows:

“[515c1] Then, in every way such prisoners would deem reality to be nothing else than the shadows of the artificial objects.”

“Quite inevitably,” he said.

“Consider, then, what would be the manner of the release and healing from these bonds and this folly if in the course of nature something of this sort should happen to them: When one was freed from his fetters and compelled to stand up suddenly (*εξαίφνης*) and turn his head around and walk and to lift up his eyes to the light, and in doing all this felt pain and, because of the dazzle and glitter of the light, was unable to discern the objects whose shadows he formerly saw, [515d] what do you suppose would be his answer

if someone told him that what he had seen before was all a cheat and an illusion, but that now, being nearer to reality and turned toward more real things, he saw more truly? And if also one should point out to him each of the passing objects and constrain him by questions to say what it is, do you not think that he would be at a loss and that he would regard what he formerly saw as more real than the things now pointed out to him?"

"Far more real," he said."

"And if he were compelled to look at the light itself, would not that pain his eyes, and would he not turn away and flee to those things which he is able to discern and regard them as more clear and exact than the objects pointed out?"

"It is so, he said." [*Republic*, Book VII (515c1-e5), translation Paul Shorey.]

As in the case of the *Parmenides* dialogue, Plato's allegory of the cave illustrates the axiomatic shock that the individual must go through when he turns his head suddenly and sees the light of day coming from outside of the cave; and then, fearful of the implications of the difference in what he had been used to look at, he prefers to go back to the illusions of the cave, and accept the fact that truth is not for him to know. The turning of the head, the going backward, and the sudden painful twisting motion of the mind are all significant in order to understand the modification process of an axiomatic change.

Similarly in *Letter Seven*, Plato identifies five degrees of knowledge of which only the fifth corresponds to true reality [*Letter Seven* (342a-343e)]. Plato's idea of *hypothesizing the higher hypothesis* is the ultimate sudden moment when the mind discovers the highest level of human thinking. However, the process of reaching this level is fraught with obstacles where the mind must go beyond the contradictions of opposites, or what Nicholas of Cusa later called, the domain of the "*coincidence of opposites*."

The idea of an axiomatic change first appears to the human mind as a strange event which Plato identified as the "starting point of two inverse directions of change" [*Parmenides* (155e9)]. The Greek word he used was *exaiphnes*

(ἐξαίφνης), meaning “suddenly,” “unexpectedly,” “without warning,” or “instantaneousness.” The root meaning of the Greek term connotes a moment of surprise or of being caught unaware all of a sudden that something is happening. In a sense, *exaiphnes* does not describe the content of an idea, but describes the *surprising moment of a change* that modifies the state of your mind. All attempts at identifying the nature of such a moment generally fail because the very act of attempting to discover it pushes it away. This also expresses a moment of exaltation (ἐξαιρέτιν) such as the flash of discovery which takes place in a perplexing yet joyful moment of revelation. In the Bible, there are five references to such an unusual and enlightening moment:

Mark 13:36 “lest coming suddenly (*exaiphnēs* / ἐξαίφνης | adverb) he should find you sleeping.”

Luke 2:13 “And suddenly (*exaiphnēs* | ἐξαίφνης | adverb) there was, with the angel, a multitude of the heavenly host, praising God and saying,”

Luke 9:39 “and a spirit seizes him, and he suddenly (*exaiphnēs* | ἐξαίφνης | adverb) cries out; and it throws him into convulsions and causes him to foam at the mouth; and with difficulty it departs from him, bruising him as it leaves.”

Acts 9:3 “As he traveled along, approaching Damascus, suddenly (*exaiphnēs* | ἐξαίφνης | adverb) a light from heaven shone all around him;”

Acts 22:6 “As I journeyed and came near to Damascus, about noon suddenly (*exaiphnēs* | ἐξαίφνης | adverb) out of heaven there flashed a brilliant light all around me.”

American philosopher, Joseph Cimasky, wrote an exceptionally insightful doctoral thesis on this subject, and identified the Platonic idea as a powerful educational principle which is able to build the character of the human individual. Cimasky wrote:

“In Plato’s hands *ἐξαίφνης* represents something akin to a flash of illumination, and this sudden illumination experience is transformative and

self-sustaining. The change that it inspires is enriching and constructive. Consequently, Plato's philosophy reorients ensuing conceptions about the sudden, and, in turn, supplants the Homeric worldview that characterized sudden change as destructive and disastrous. [...] My intent is to demonstrate both the significance of *ἐξαίφνης* in Plato's philosophy and that following Plato, *ἐξαίφνης* and its cognates often came to represent the peak of philosophical enlightenment, divine revelation, or conversion experience.”⁶

The point, here, is not to focus on the religious or mystical conversion experience which this adverbial action also conveys, but to note that what Plato describes extensively in the allegory of the cave (*Republic*, VII), in his *Parmenides*, his *Letter Seven*, as well as in the vision of the “Beautiful” in his *Symposium*, pertains directly to what Lyndon LaRouche developed as the enlightenment that the mind acquires when making a discovery of principle of the transfinite domain, similar to Paul's discovery on the road to Damascus (Acts 22:6). For Plato, the concept reflects an experience of the mind's ability to go beyond the apparent finite limitations of sense perception. This experiment is the actual proof that there are no limits to growth.

What happens to your mind when it goes through such an axiomatic change? In the third hypothesis of his *Parmenides* dialogue (155e -157b), Plato investigates what happens to the mind when it changes from a lower to a higher domain of thinking. LaRouche would have said: “when the mind goes from a lower to a higher manifold.” There is a certain degree of playfulness, here, which should not be missed. This is how Plato examines the question:

“Will the One not be in some strange state at the moment it changes?”

“Which strange state is that?”

“An unexpected instantaneousness (*exaiphnēs* - *ἐξαίφνης*). That is, in fact, what seems to be the meaning of suddenness (*exaiphnēs* - *ἐξαίφνης*); that is,

⁶ Joseph Cimasky, [*The Role of Exaiphnes in Early Greek Literature: Philosophical Transformation in Plato's Dialogues and Beyond*](#), Lexington Books, Lanham, MD, 2017. See PDF copy: [*All of a Sudden: The Role of 'Εξαίφνης in Plato's Dialogues*](#).

a sudden starting point between two inversed states of changing directionalities. Because it is not from the non-moving immobility that change is able to surge; nor is it from the motion moved by the transition of the change. It is rather in the strange nature of the instantaneousness (*exaiphnēs* - ἐξάιφνης), an inbetweenness which, located outside of time in the interval between mobility and immobility, is precisely and simultaneously the point of departure and the point of arrival for the change which passes from mobility to rest and from rest to mobility.

“That has every chance to be true.

“Thus, since the One is both in an immobile state and in motion, it will have to change in order to go from one state to the other: it is only under this condition, in fact, that it can compose with both states. That is, this operation of change can only take place in a sudden instantaneousness (*exaiphnēs* - ἐξάιφνης); and while it changes, it cannot partake of any moment of chronological time, no more than it could be moved or be unmoving.”

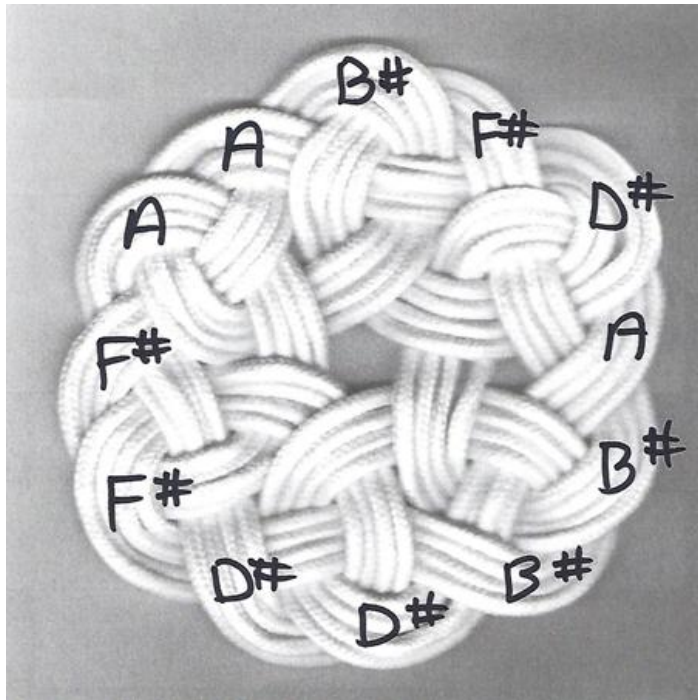
[*Parmenides* (155d1-156e11) P. B. translation]

It is only after the discovery of this instantaneousness that Plato realizes that everything in his mind has been changed and reordered differently and that new axioms have all of a sudden been introduced from a higher domain of thought which is beyond contradictions. This is the closest that Plato comes to identifying that the nature of this instantaneous event of the One is transfinite. This One singular event does not exist in chronological time, but in the simultaneity of eternity; and since it partakes of two axiomatically different domains, past and future at the same time, it is as if it is being created between two different manifolds and it describes the creative process of change itself, as in the *coincidence of opposites* of Cusa's maximum circle and infinite straight line, (see figure) or as the passing from the ellipse to the parabola in the Leibniz case (see figure).

The best visual metaphor I could compose to illustrate the strangeness of the process that Plato describes is the multiple twisting of a single knot, as a One over

the Many, expressing, for example, the Lydian principle that Beethoven used in composing his *Sonata Quasi Una Fantasia*, Opus 27, No. 2.

Follow the pathway of this knotwork with your finger by going two steps forward and one step back, starting your circular action clockwise at one o'clock



The motion first starts at **F#** at one o'clock, then goes down to **B#** at 4 o'clock, then back up under **D#** to **A** at 3 o'clock. Then, tie the four Lydian knots into a single One: [**F#, B#, A**], [**D#, B#, F#**], [**D#, A, F#**], [**B#, A, D#**]. *The One tone that is not there*, but which you can hear coming into your mind from the future, is the key of **C# Minor**, which is the One that provides the unity of Beethoven's composition; that is, the sudden One over the Many.

Knotty Lydian intervals for Beethoven's Piano *Sonata quasi una fantasia*, No. 2, Opus 27, measure 35.

In *Letter Seven*, Plato expressed that sudden effect of this mental experience as a pleasant and profound experience of the transfinite; that is, as an epistemological discovery of a higher universal friendship, an experience similar to that of St. Paul's *agape* that Leibniz later identified as the Republic's principle of the "Pursuit of Happiness." Plato wrote:

"One statement, at any rate, I can make in regard to all who have written or who may write with a claim to knowledge on the subjects to which I devote myself – no matter how they pretend to have acquired it, whether from my instruction or from others or by their own discovery. Such writers can in my opinion have no real acquaintance with the subject. I certainly have composed no work in regard to it, nor shall I ever do so in future, for there is

no way of putting it in words like other studies. Acquaintance with it must come rather after a long period of attendance on instruction in the subject itself and of close companionship, when suddenly (*exaiphnēs*), like a blaze kindled by a leaping spark, it is generated in the soul and at once becomes self-sustaining.” [Letter Seven (341b9-d2)]

The subject in question represents the transfinite nature of *hypothesizing the higher hypothesis* which Plato addressed to his friend Dion in the hope that Dionysius would discover his ability of becoming the philosopher king that he was trying to get him to become. Plato identified this unspeakable sudden transformation as an axiomatic moment of change, which LaRouche has identified as a transfinite moment, a moment where a new principle of discovery is acquired at the highest level of thinking. Plato identified it as the fifth and highest degree of knowledge:

“For everything that exists there are three classes of objects through which knowledge about it must come; the knowledge itself is a fourth, and we must put as a fifth entity the actual object of knowledge which is the true reality. We have then, first, a name, second, a description, third, an image, and fourth, a knowledge of the object. Take a particular case if you want to understand the meaning of what I have just said; then apply the theory to every object in the same way. There is something for instance called a circle, the name of which is the very word I just now uttered. In the second place, there is a description of it which is composed of nouns and verbal expressions. For example the description of that which is named round and circumference and circle would run as follows: the thing which has everywhere equal distances between its extremities and its center. In the third place, there is the class of object which is drawn and erased and turned on the lathe and destroyed – processes which do not affect the real circle to which these other circles are all related, because it is different from them. In the fourth place there are knowledge and understanding and correct opinion concerning them, all of which we must set down as one thing more that is found not in sounds nor in shapes of bodies, but in minds, whereby it evidently differs in its nature from the real circle and from the

aforementioned three. Of all these four, understanding approaches nearest in affinity and likeness to the fifth entity, while the others are more remote from it.

[...]

“Hardly after practicing detailed comparisons of names and definitions and visual and other sense perceptions, after scrutinizing them in benevolent disputations by the use of question and answer without jealousy, at last, in a flash, understanding of each blazes up, and the mind, as it exerts all its powers to the limit of human capacity, is flooded with light.” [*Letter Seven* (342a9 – d2 [...] 344b4 – 10 [...] 344d2)]

Finally, Plato's discovery is similar to the Christian discovery of the idea of brotherhood in St. Augustine's *City of God*, which is realized by spreading brotherly love (*agape*) among all of the peoples of the world, which has been attempted historically four times during the last two thousand years; the first took place with the Apostles during and after Christ; the second with Joan of Arc and Louis XI, which was brought to success with Nicholas of Cusa with the Brotherhood of the Common Life through the Italian Renaissance; the third attempt was made by Leibniz with the creation of the Academy of Arts and Sciences; and subsequently, a fourth attempt was made by Lyndon LaRouche with the institution of the International Caucus of Labor Committees (ICLC). The immediate future ahead will tell if this last Platonic brotherhood of mankind will succeed in taking hold. In all events, it was Dante who best expressed the magnitude of such a Platonic experiment in the conclusion of his *Divine Comedy*:

“Like a geometer, who sets himself to measure, in radii, the exact circumference of the circle, and who cannot find, by thought, the principle he lacks, so was I, at this new sight: I wished to see how the image fitted the circle, and how it was set in place, but my true wings had not been made for this, if it were not that my mind was struck by lightning, from which its will emerged.

“Power, here, failed the deep imagining: but already my desire and *will* were rolled, like a wheel that is turned, equally, by the Love that moves the Sun and the other stars.”⁷



Dante's Paradise by Gustave Doré.

⁷ Dante, *Paradise*, Canto XXXIII. Translated by A. S. Kline © Copyright 2000. All Rights Reserved.

THE EPISTEMOLOGICAL SIGNIFICANCE OF CUSA'S LEARNED IGNORANCE

Lyndon LaRouche often emphasized the necessity of solving the problem of squaring the circle as a means of advancing the general domain of human knowledge. His choice example was always to show how to go from a lower to a higher manifold by means of circular action, as opposed to by the failed attempts of linearity in the small. LaRouche was able to solve this problem by constructing the epistemology of what he termed “the fourth level of mathematics”⁸ through the successive works of Cusa, Leibniz, Riemann, and Cantor. LaRouche went to the core of this problem with reference to Cusa:

“Nicholas of Cusa’s discovery wasn’t the linear extension of Archimedes. It was a rejection of Archimedes, and a discovery of the fact that circular action is ontologically superior to polygonal action, because it’s not linear. For instance, if you have a circle you can create a square by folding the circle twice. And you can create other polygonal figures within a circle. But you cannot go from a polygon to creating a curved circumference. So the circle is actually transcendental in relationship to a polygon. And that was a completely new discovery! It had never existed in human history before.

“And that’s the quality of thinking that’s actually required: You find that in Einstein, you find that in some other individuals. And that quality is really what we have to make clear to people, this is what really makes

⁸ Lyndon LaRouche, [*LaRouche in dialogue with Russian science*](#), EIR, Vol. 21, No. 24, June 10, 1994, p. 35. “The third level of mathematics was discovered approximately 1 440 A.D. in Florence, Italy, by Cardinal Nicolaus of Cusa. This discovery forms a central descriptive feature of his famous *De Docta Ignorantia*, and was then described in some more detail formally in 1453, in a second paper called *De Circuli Quadratura*. In *De Circuli Quadratura*, Cusa says, ‘I have discovered a higher species of mathematics.’ Today we call that the mathematics of transcendental functions.

“The fourth level of mathematics was probably discovered first by Leibniz. It is the subject of his famous *Monadology*. This level of mathematics was later expanded during the nineteenth century by the successive work of Gauss, Dirichlet, Riemann, Weierstrass, and so forth, and then was finally represented, systematically, in a series of papers concluding in 1897 with the *Contributions to the Development of a Theory of Transfinite Numbers* by Georg Cantor.”

people human, and this is what we need to advance humanity, this kind of thinking.”⁹

What Cusa did was to move his attention away from the circular perimeter of a polygon to paying attention to circular action itself; and if you do the same, as Leonardo da Vinci did, in his work on *The Divine Proportion* with Pacioli and his studies on knots, you will actually be able to go from the lower domain of two dimensions to the higher domain of the third conical dimension.



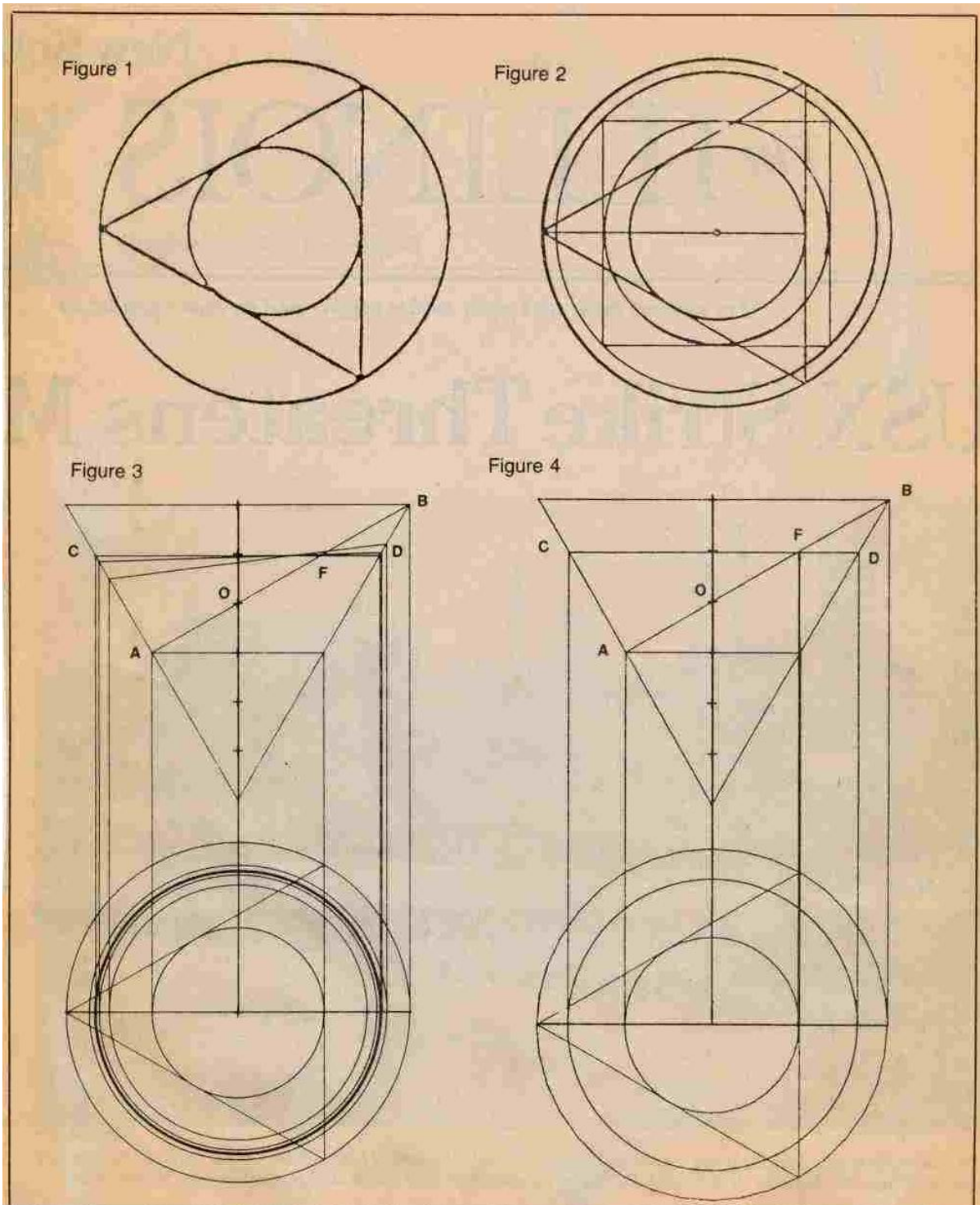
The coincidence of opposites between the finite circle and the infinite circle by Nicholas of Cusa. Photo by Antony and Susan DeFranco

⁹ Lyndon LaRouche, [*Lyndon LaRouche Is the Soul Of the United States of America*](#), EIR, Vol. 43, No. 38, September 16, 2016, p. 24.



The isoperimetric circle by Nicholas of Cusa. Photo by Antony and Susan DeFranco

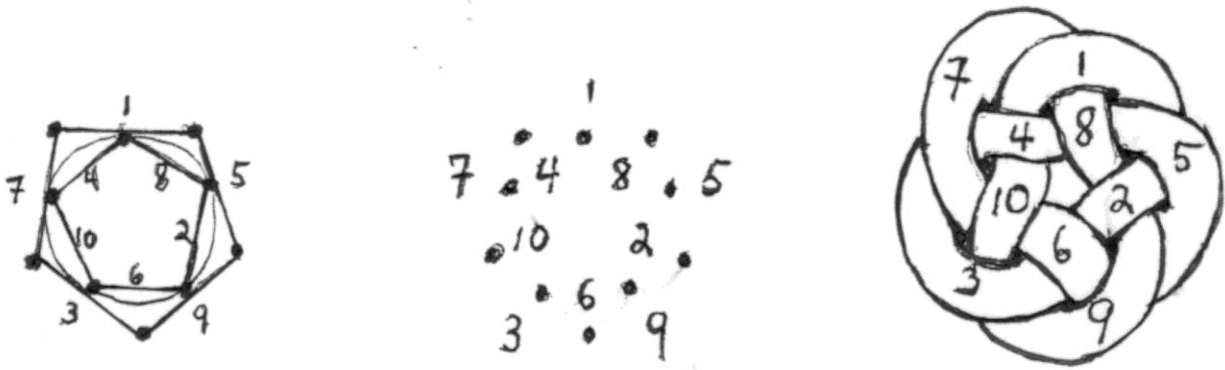
LaRouche indicated that this problem leads to what Leibniz recognized as the “*continuum paradox*” and to the axiomatic requirement that Bernard Riemann discovered in his habilitation dissertation, both of which come down to the fact that “we must leave the domain of Mathematics and go to the domain of Physics” or leave the domain of geometry to go to the domain of epistemology. In terms of national governing, this is the equivalent of saying that one must leave the domain of politics and go to the domain of constitutional moral truth.



My conical-geometrical solution to Nicholas of Cusa's isoperimetric circle. Note how Figure 3 locates in the conical-elliptical cut **AB** between the inscribed and circumscribed circles of the equilateral triangle.

Thus, a conical spiral projection, or a torus spiral projection, is the best means of capturing the epistemological significance of Cusa's isoperimetric discovery, not simple circular action as such. If you increase the number of ellipses between the sides of two series of continuously increasing polygons, and project them as they are illustrated above in Figures 2 and 3, you will reach the limit of an elliptical series where the last ellipse, **AB**, will pass into and coincide with the isoperimetric circle **CD**.

The difference between the two domains of the conical-elliptic-function and of the circle is not mathematical; it is epistemological and the *inversion method of epistemological measurement* is the only real measure that can deal with the apparent discontinuities located in the lower two-dimensional domain. LaRouche asked the question: "What are the physical measurements of a mental act of fundamental discovery? Let's just ask one more question in this connection, and pose one more Socratic question: What is the most effective way of educating a child?"¹⁰ Ask a child what he thinks of this following transformation:



Non-linear axiomatic transformation between polygon, circle, and torus circular action.¹¹

What happens in the above illustration is a change in axioms from the second to the third dimension; that is, a change which overturns the underlying assumptions which dominate the discontinuous relationship between the polygons and the circle, as well as the entire domain of simple circular action. Now, the rules of the game have been changed. We no longer use polygons, circles, and simple

¹⁰ Lyndon LaRouche, [LaRouche in dialogue with Russian science](#), p. 41.

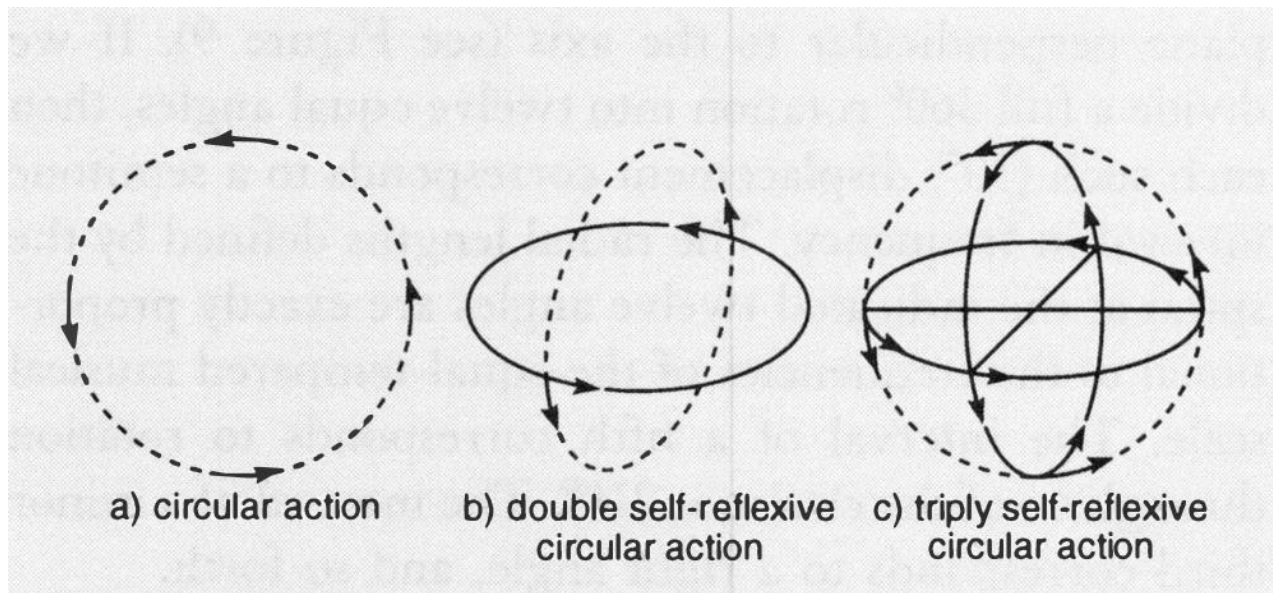
¹¹ See my 1996 class-video on [TIME REVERSAL](#).

circular action; we use doubly and triply-connected conical and toroidal spiral action as the generative principle of the new higher domain of the cone, the torus or the sphere. Those are the new axioms of conical and toroidal spiral action.

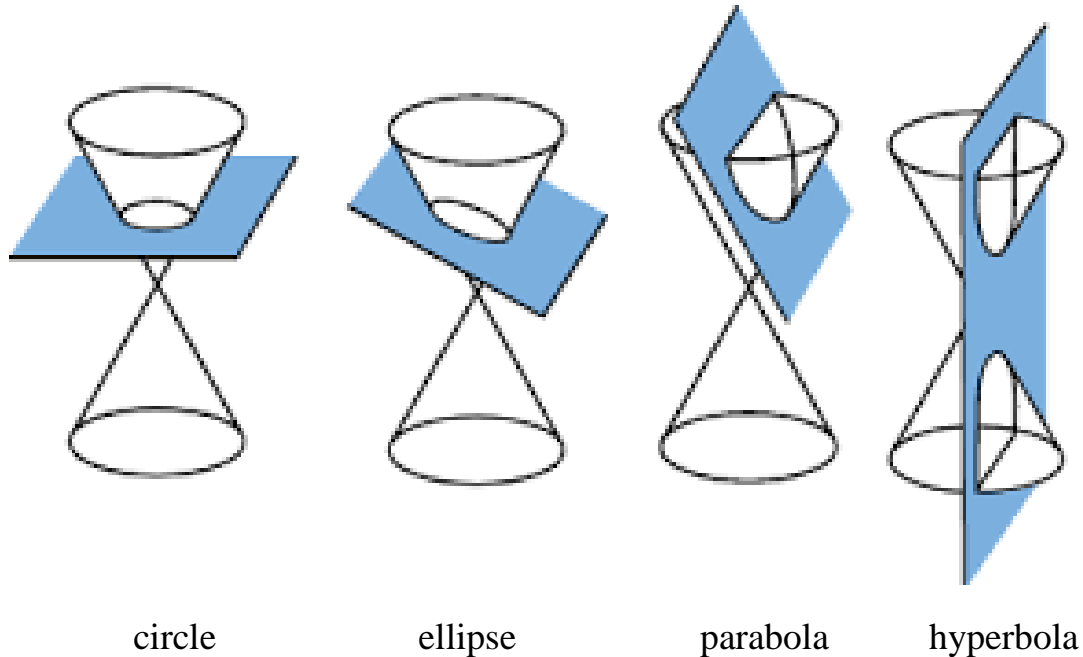
THE EPISTEMOLOGICAL MEASURE OF LEIBNIZ'S PRINCIPLE OF CONTINUITY

Gottfried Leibniz may be the only philosopher in all of history to have identified the moment of change from one species to another with total geometrical precision. Although the nature of the change may be located within the single domain of conical functions, it appropriately describes the singularity of passing from one geometrical species of curves to a different species of curves; while emphasizing, at the same time, the continuity principle which dominates the entire process as if it were an axiomatic change.

Take the case of the three dimensional domain as an example. Plato initially identified the three dimensional domain of space as being expressed by three different circular planes and six different directions of spherical motion. The closure of Platonic space is therefore spherical and can be represented, as being inter-connected at right angles, with three planes of circular action.

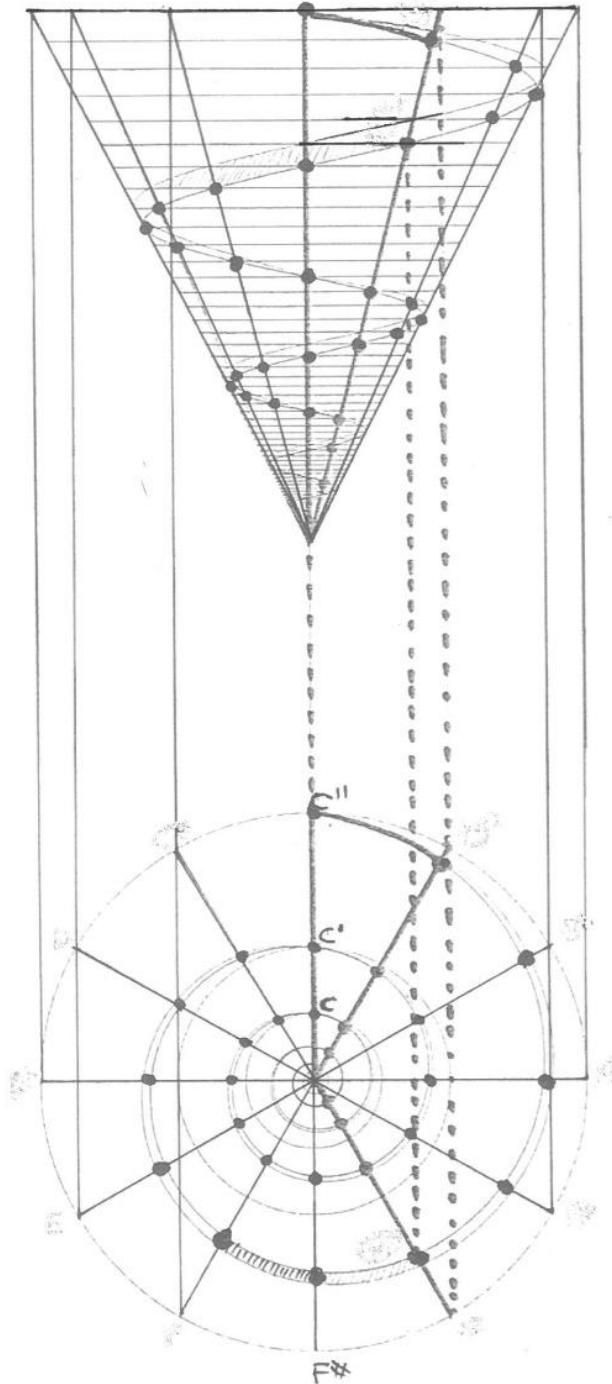


Similarly, the most elementary means of getting closure with the cone is by generating the four basic conic sections, circle, ellipse, parabola, and hyperbola, by continuously rotating a plane across a cone at a continuous angle of ninety degrees. As Leibniz noted, these four conical sections are totally discontinuous in the two dimensional domain, but are continuous within the three dimensional domain.



Furthermore, each conical section can also be represented as a member of the same conical family by means of an orthographic projection. A static conical projection can also be created as an orthographic projection demonstrating the arithmetic and geometric means of a logarithmic-conical-spiral-action as they are projected onto the base-circle of a flat spiral action of equal-logarithmic-tempering. The equal-tempered notes of the two octaves are marked with dots on both the logarithmic three dimensional conical spiral and on the flat two dimensional spiral of the circular plane. This is how the musical system of equal-tempering can best be geometrically represented in both domains at the same time. Any other form of three dimensional projections is a mere approximation. (See figure below)

Leibniz's insight into such a logarithmic orthographic conical projection is crucial for understanding our subject, in that he identified the epistemological significance of the continuity of the geometrical change in curvature which takes place between the ellipse and the parabola, in order to show that the limits of transformation from one into the other is a heuristic metaphor of what occurs inside of your mind without a leap when an axiomatic change takes place.



The case Leibniz used to demonstrate his *principle of continuity* is the transformation of an ellipse into a parabola. In a letter dated July 1687, Leibniz wrote to Malebranche:

“This principle has its origin in the *infinite* and is absolutely necessary in geometry, but it is effective in physics as well, because the sovereign wisdom, the source of all things, acts as a perfect geometrician, observing a harmony to which nothing can be added. This is why the principle serves me as a test or criterion by which to reveal the error of an ill-conceived opinion at once and from the outside, even before a penetrating internal examination is begun. It can be formulated as follows.

Logarithmic-conical-spiral projection of two musical octaves

“When the difference between two instances in a given series or that which is presupposed can be diminished until it becomes smaller than any given quantity whatever, the corresponding difference in what is sought or in their results must of necessity also be diminished or become less than any given quantity whatever. Or to put it more commonly, When two instances or data approach each other continuously, so that one at last passes over into the other, it is necessary for their consequences or results (or the unknown) to do so also. This depends on a more general principle: that , as the data are ordered, so the unknown are ordered also. [*Datis ordinatis etiam quaesita sunt ordinate.*]

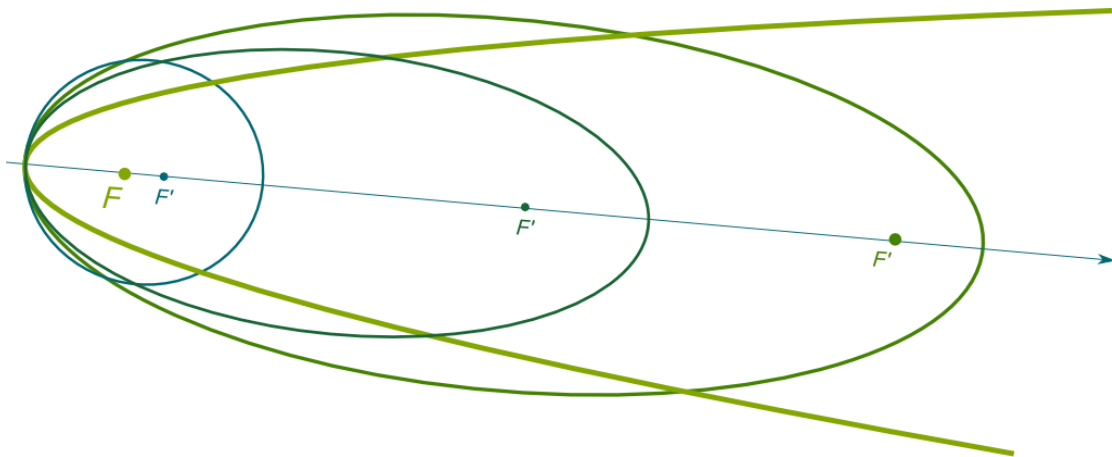
“But examples are needed in order to understand this. We know that a given ellipse approaches a parabola as much as is wished, so that the difference between ellipse and parabola becomes less than any given difference, when the second focus of the ellipse is withdrawn far enough from the first focus, for then the radii from that distant focus differ from parallel lines by an amount as small as can be desired. And, as a result, all the geometric theorems which are proved for the ellipse in general can be applied to the parabola by considering it as an ellipse one of whose foci is infinitely far removed from the other, or (to avoid the term infinite) as a figure which differs from some ellipse by less than any given difference.”¹²

The lesson to be learned, here, is that every time your mind goes harmonically and continuously to infinity, it will go through an axiomatic change. These are the two fundamental characteristics of Leibniz’s idea of a substance or of a monad: *harmony and continuity*.

What characterizes each monad, individually, is its ability to continuously change without discontinuous finite parts; in that sense, a monad is a simple self-subsisting substance as a whole, such as a human mind, which is capable of subsisting as a subject or a self-subsisting continuum with internal capabilities for

¹² Gottfried Leibniz, *Philosophical Papers and Letters*, Edited by Leroy E. Loemker, Kluwer Academic Publishers, Vol. 2, Boston, 1989, pp. 351-52

dynamic axiomatic transformations. Change inside of the monad is continuous and always takes place without jumps or gaps of discontinuity. However, even when our mental perception of such changes appears to be discontinuous, the changing of manifolds inside of the mind remains continuous and harmonically infinite.



As the second focus goes to infinity, ellipses become a parabola

Five years later, Leibniz wrote his last critical thoughts against Descartes which completed his monadology doctrine on this subject of change. One last time, Leibniz used the elliptic-parabolic change to close this chapter on this crucial axiomatic subject:

“I usually call this the *law of continuity*. I have already explained this principle elsewhere, but it must be repeated and amplified here. When two hypothetical conditions or two different data continuously approach each other until the one at last passes into the other, then the results sought for must also approach each other continuously until one at last passes over into the other, and *vice versa*. For example, if one focus of an ellipse remains fixed and the other recedes farther and farther away from it, while the *latus rectum* remains constant, the new ellipses which thus come into being continuously approach a parabola and finally pass over into it completely, namely when the distance of the receding focus becomes immense. Therefore the properties of these ellipses must also approach more and more the properties of a parabola until at last they pass into them, and the parabola can be considered as an ellipse whose second focus is infinitely distant. All the properties of an ellipse in general will thus be found in the parabola

considered as such an ellipse. Geometry is full of examples of this kind, but nature, whose most wise Author uses the most perfect geometry, observes the same rule; otherwise it could not follow any orderly progress.”¹³

The point here is to have an “orderly progression.” When Plato used the concept of instantaneousness (*exaiphnēs*), he did not imply that change occurred through a leap; he implied, as later did Leibniz, that an axiomatic change takes place as a sudden and instantaneous inversion where two different dimensional degrees of the mind fold into each other as if into a higher single new domain of the One over the Many.

Moreover, there is an underlying assumption that Leibniz brings out and which is to be considered when comparing a mental and a physical axiomatic change. In the case of a physical axiomatic change, “*there is never an action of bodies without reaction and that both are equal to each other and in contrary directions.*”¹⁴ The physical action/reaction here is not the same as in a mental process. Although British geopolitics would have you think the contrary, it is the continuous (i. e. infinite) power of mind which prevails over the power of reaction and which becomes reinforced through a series of higher mental axiomatic changes which supersede physical action/reaction. In his *Specimen Dynamicum* of 1696, Leibniz added the following further clarification:

“It is also in agreement with this *law of continuity*, which excludes a leap from change, that the case of rest can be considered as a special case of motion, namely, the case of a disappearing or minimal motion, and that the case of equality can be held for a case of disappearing inequality. The consequence is that the laws of motion must be set up in such a way that particular rules are not necessary for equal and resting bodies, but that these arise from the rules for unequal and moving bodies as such.”¹⁵

The point is that it is *the continuity of the infinite* which prevails over the finiteness of the physical domain, because it is only the force of determination of

¹³ Gottfried Leibniz, *Op. Cit.*, pp. 397-98.

¹⁴ Gottfried Leibniz, *Op. Cit.*, p. 449.

¹⁵ Gottfried Leibniz, *Op. Cit.*, p. 447.

the free will, and not the effects arising from a physical conflicting encounter, which truly exists among human relationships; therefore, it follows that power of mind becomes self-sufficient when at peace with mankind; that is, when constantly connected to the infinite and the common good; and that is the reason why the mind and the body are so harmonically well-ordered to one another, and that it is the mind which must always rule the body.

LAROCHE HYPOTHESIZING THE PRINCIPLE OF THE CREATIVE PROCESS

The emotion involved in the Chartres Cathedral's sculpture of *God*



Contemplating Adam in His Mind carries within it nothing short of such a transfinite emotion of the creative process that pertains to the human will. The question of such *forethought* is as follows: "What is the axiomatic assumption which underlies the argument whereby Man is created in the Living Image of God?" The assumption is that God is good because he created man to be in charge of making changes in the universe as the best possible world that can be. Thus, what is represented in this Cathedral statue is not the bodily image of God and man, but the reference to the individual human mind and its creative potentiality.

Chartres Cathedral North Porch.

Such a potentiality includes the imaginative state of the human intellect made to discover higher forms of mental existence, which never existed before,

and through which man can master the universe, but only under the condition of universal lawfulness. Such is the true representation of the meaning of being in the Living Image of God.

Here, the backward and forward motions of this representation of the creative process cause the relationship of God and man to generate a major dissonance; that is, the fact that God is both unfolding and enfolding His Idea into a single motion that projects into the future what is in the back of His Mind, as if time reversal were to our human minds what the simultaneity of eternity is to God's Mind. This is the same ambiguity that Plato located in his idea of *exaiphnēs* in the *Parmenides*, or the instantaneous interval of simultaneity of eternity in an axiomatic change going in two directions at the same time. Here, LaRouche refers us back to Plato's *Parmenides* by identifying the four different mathematical transfinite levels of hypothesizing:

“To illustrate the meaning of the term *higher hypothesis*, reference the given list of the four general levels of cardinality (“power”) in mathematics. Looking back across the internal history of mathematics from the vantage-point of Cantor's higher transfinite orderings, the succession of axiomatic-revolutionary changes defining the succession *rational, algebraic, transcendental, Alephs* mathematical types of cardinalities is derivable by a constant method of hypothesis-making. So, in the language of Plato's *Parmenides*, the conception of this type of constant method of hypothesis-making is a *One*, relative to the four *Many* (the four types of cardinalities).”¹⁶

To simplify the argument, here, let the unity of succession of hypothesis represent a higher hypothesis; similarly, let the unity of succession of several higher hypotheses represent the hypothesizing of the higher hypothesis. LaRouche explained the essentials of this transfinite process as follows:

¹⁶ Lyndon LaRouche, [*The Fraud of Algebraic Causality*](#), Fidelio Magazine, October 3, 1994. LaRouche noted: “Just as simple hypothesis, expressed as an interdependent set of axioms and postulates, defines the principle of deductive consistency, so a principle of generation of a type of hypotheses, higher hypothesis, defines a higher, governing “consistency” among all members of array (lattice) of that type. Thus, the combination of Euclidean and all non-Euclidean formal geometries is a Many subsumed by a subsuming principle of purely constructive geometry, a principle which subsumes all possible formal geometries developed in the same axiomatic-revolutionary way.”

“The formalist state of mind is obsessed with method of formal proof, formal consistency with a set of underlying, axiomatic assumptions. Creative discovery signifies overthrowing some of those axiomatic assumptions; for such a case a formal proof is not possible. The person who does not immediately recognize the empirical distinction between the two distinct species of thinking, is neither a scientist nor a competent policy-shaper or other professional in the field of education.

“The student advantaged to enjoy such a Christian humanist mode of secondary education, thus locates knowledge not in mere ‘facts,’ but in the process of generating knowledge within those creative processes which are empirically defined for that student by the repeated reliving of the moments of valid discovery by original discoverers. That student, by the time he or she is graduating from such an institution, can recognize readily the significance of Plato's term *hypothesis*. He or she can recognize those kinds of discovery achieved through overturning previously held axiomatic assumptions: valid such discoveries are Platonic *hypotheses*. Similarly, once the student comprehends individual *hypothesis* in this mode, the student is able to employ the method of *hypothesis* to define the higher One subsuming a large array of individual valid, axiomatic-revolutionary discoveries (*hypotheses*). All of the discoveries which, as a (e.g., transfinite) series are generated by a common (higher) *hypothesis* respecting the method of generating such discoveries, are a Platonic Many commonly subsumed by a Platonic One.”¹⁷

Here is how LaRouche proposed to discuss the Unifying power of the mind with the resolving of the paradox of the quadrature of the circle:

“One might begin the classroom blackboard exercise with a circle and a pair of respectively inscribed and circumscribed squares. Next, double repeatedly, at an equal speed, the number of sides of each of these respectively inscribed and circumscribed polygons. At that point in the

¹⁷ Lyndon LaRouche, [*The Truth About Temporal Eternity*](#), FIDELIO Magazine, March 14, 1994.

lesson, our attention must be turned to the famous "method of exhaustion" associated with a mathematician of Plato's Academy of Athens, Eudoxus.

“Let the class ask itself: What is the relationship between the circular perimeter and the perimeters of the polygons when the n of 2^n becomes extremely large? Focus upon two adjacent sides of the inscribed polygon at that instant of the ongoing process, as if in a suitably powerful microscopic enlargement. Examine the relationship between the two polygonal perimeters in that vicinity, and the segment of circular perimeter lying between them. Extend the process to a value of $2^{(n+n)}$. Repeat the microscopic scrutiny. Extend the process to the degree that a polygonal side the length of one micron would require a circle larger than the currently imagined largest size of our universe. It changes, but it remains the same: the polygonal species and the species responsible for the existence of the circle can never become congruent.

“At this point, the Classical scholar must recognize that this problem of quadrature has affinities with Plato's *Parmenides*. It appears that the circular action, which both generates the circle and is crucial for constructing the polygonal series, defines and bounds externally all the polygons of this series, but can never be a member of the series which it defines in a subsuming way.

“At this juncture in the experiment, the student might pick up his drawing compass, studying it very thoughtfully: This compass has no place to exist within the set of axioms and postulates of what we term Euclidean geometry! This Archimedean construction which we followed so faithfully has a terrible error of assumption built into it, at least as that theorem has been ordinarily presented in schools. The act of *circular rotation*, which defines and bounds the polygonal series, is not allowed within the set of Euclid's ontologically axiomatic notions of point, and straight line as a "shortest distance between two points." The latter set belongs to the domain of mere space; *circular action* belongs to the domain of *space-time*—as Johann Bernoulli and Gottfried Leibniz proved the latter in 1697, when they

established non-algebraic mathematical physics, and did so, on the basis of the physical-geometrical principles of refraction of radiated light. Some of the deeper implications of this for mathematical physics awaited those fundamental discoveries which Georg Cantor presented two centuries later, in 1897.”¹⁸

Again, the ability of the human mind to go beyond the singularity or the discontinuity between enfolding and unfolding or between the polygon and the circle is the simplest and most effective way to go back to break the limitations of the linear and finite domain and reach forward into the higher transfinite domain of multiply-connected circular action. The change, here, calls for an axiomatic transformation whereby the mind must refuse to accept the fallacious assumption that the curvature of the circle can be reached by any such additional discontinuous steps. The increasing of the inscribed and circumscribed polygonal perimeters will never coincide with the circle. That is nothing but a magical trick that some retarded professor has pulled out of some dusty old Aristotelian flat hat.

No such sense deception is possible, because in truth, the polygon and the circle represent two completely different geometrical domains, where one has nothing but discontinuities and the other is everywhere constantly continuous. At this point, Plato would have suddenly added what LaRouche pointed out, which is: “To a scientific mind, that construction proves that never can the two coincide, because they represent different species of existence. In the domain of mathematical physical science, that quality of Socratic negation is the onset of a creative mental act of axiomatic-revolutionary discovery.”¹⁹

Thus, the knowledge that this quadrature cannot take place requires a moment of *learned ignorance* where the mind must conclude that some other condition must be set in order to go beyond that limit. That is where the apparent gap resides. LaRouche further explained what that new condition should be:

¹⁸ Lyndon LaRouche, [*The Truth About Temporal Eternity*](#).

¹⁹ Lyndon LaRouche, [*The Truth About Temporal Eternity*](#).

“This leads to a further step. If we avoid the trap of reading the word "halving" in an empty, arithmetic way, we are obliged to examine the construction by means of which the series 2^n might be generated in visual and further-extended space-time. The construction itself is bounded by circular *action*. The proposition must be restated accordingly: The possibility of generating indefinitely the series 2^n depends upon circular *action*; *circular action* is thus the crucial feature of the *generating-principle* of construction of the *transfinite* series of polygons, both the respectively inscribed and the circumscribed series treated as a single series. Thus, the same quality of *circular action* which bounds the inscribed series *externally* and the circumscribed series *internally* also determines the generating principle of both series, and, in that sense, bounds the combined series *externally*, from outside and above the set of axioms and postulates upon which a Euclidean geometry of simple space depends for all its consistent theorems.

“Thus, creative mentation concludes, the difference between the species of polygons in Euclidean space and circular action is an ontological difference; therefore, the use of Archimedean construction to approximate a circular perimeter by averaging the difference between the two polygonal 2^n series, prompts the eruption to view of an underlying *ontological paradox*. The species of circular perimeter cannot be generated honestly as a theorem from the set of axioms and postulates of formalist Euclidean space. Thus, the two species are distinct.”²⁰

CONCLUSION

Plato's discovery of instantaneousness (*exaiphnēs*) raises the level of Socratic irony to a higher threshold by bringing out the example of the flaw in the political deception that is called “democracy.” Socrates perfectly identified the highest democratic result of such a man of “equality” as follows:

²⁰ Lyndon LaRouche, [*The Truth About Temporal Eternity*](#).

“Furthermore, said I, he spends all of his days in this fashion, indulging into his proclivity of the moment, now getting drunk and abandoning himself to the lascivious sounds of the flute; and the next moment, drinking only water and dieting; sometimes he exercises his body and at some other moment, he just sits around without a single concern in the world. Sometimes he will even occupy himself by taking up philosophy. More often than none, he will try out his talents in politics, just to jump up and down from his seat to say and do whatever comes to his mind. And, if he happens to admire military men, he is pulled in that direction, and if money making attracts him, he turns to that side. In other words, there is no order and no restriction to his behavior. For him, [Democracy] is a totally free and pleasurable regime, and he is totally committed to cling to it to the very end.” [*Republic*, Book VIII (561cd)]

There is a profound irony, here, because how can a man go wrong when he believes in a regime which can only bring him pleasure? Thus, what must now become obvious is the fact that the world strategic situation is like the quadrature of the circle which poses a profound moral question. Why do I have to worry about the rest of the world when I can get all of the pleasures I want, when I want them?

Why? Because Plato and LaRouche established beauty and truth as a higher good that your mind craves for. You are looking , here, at a higher moral standard for science, artistic composition, and statesmanship, higher than the domain of practical deductive accommodations, higher than lying political party differences; and that is precisely the reason why multiply-connected circular action is ontologically superior to the back and forth paradoxical nature of the linear two dimensional clash between political parties, as between polygons and circles, just as peace and mutual development are superior to war and destruction.

With creativity, there is never any real deductive division in the mind; all knowledge is based on the morality of advancing the common good of mankind. Therefore, by its very nature, the human mind has the power to go beyond such limitations as does the solution to the paradox of the quadrature of the circle;

otherwise there is no justice. From that vantage point, Plato's *exaiphnēs* is the moral receptacle of universal justice.

When, for example, politics denies the individual human mind access to legitimate creative powers, as Plato's allegory of the cave showed, then humanity suffers unjustly; but, there is a higher injustice than the injustice done to any individual; there is the injustice done to justice itself. For Plato, this higher level corresponds to the level of the personal transformation through an axiomatic change, which elevates the mind to truth itself. As LaRouche put it: "The generation-principle which is a higher species than any member of the theorem-set of a transfinite ordering, stands ontologically outside and above each and all members of the set."²¹ That must become the only rule of conduct in the universe, today.

FIN

²¹ Lyndon LaRouche, [*The Truth About Temporal Eternity*](#).